Dual problem

⇒ When primal is in standard form

Consider the following LPP is in standard form

$$\text{Max } z = \sum_{j=1}^{n} c_j x_j$$

$$\text{s.t } \sum_{j=1}^{n} a_{ij} x_j = b_i \quad , i = 1, 2, \dots, m$$

$$x_j \geq 0 , \; j = 1, 2, \dots, n$$

The dual becomes

$$\text{Min } z^* = \sum_{i=1}^{m} b_i y_i$$

$$\text{s.t } \sum_{i=1}^{m} a_{ij} y_i \geq c_j \quad , j = 1, 2, \dots n$$

y_i unrestricted in signs for all i.

ues Find the dual of the primal problem

$$\text{Max } z = 5x_1 + 6x_2$$

s.t.

$$x_1 + 9x_2 \leq 60$$

$$2x_1 + 3x_2 \leq 45$$

$$5x_1 - 2x_2 \leq 20$$

$$x_2 \leq 30$$

$$x_1 , x_2 \geq 0$$

Let y_1, y_2, y_3 and y_4 be the dual variables

$$\text{Min } z^* = 60y_1 + 45y_2 + 20y_3 + 30y_4$$

s.t.

$$y_1 + 2y_2 + 5y_3 \geq 5$$

$$9y_1 + 3y_2 - 2y_3 + y_4 \geq 6$$

$$y_j \geq 0 , \; j = 1, 2, 3, 4$$

On other hand consider the primal problem

$$\text{Max } z = \sum_{j=1}^{n} c_j x_j$$

s.t.

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad , i = 1, 2, \dots m$$

x_j unrestricted in sign

Then dual problem becomes

$$\text{Min } z^* = \sum_{i=1}^{m} b_i y_i$$

s.t. $$\sum_{i=1}^{n} a_{ij} y_i = c_j \quad , j = 1, 2, \dots n$$

$$y_i \geq 0 \, , i = 1, 2, \dots m.$$

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Q Find the dual of the problem

$$\text{Max } z = 5x_1 + 12x_2 + 4x_3$$

s.t. $$x_1 + 2x_2 + x_3 \leq 5$$
$$2x_1 - x_2 + 3x_3 = 2$$
$$x_1, x_2, x_3 \geq 0$$

$$2x_1 - x_2 + 3x_3 \leq 2$$
$$2x_1 - x_2 + 3x_3 \geq 2$$

$$2x_1 - x_2 + 3x_3 \leq 2$$
$$-2x_1 + x_2 - 3x_3 \leq -2$$

$$\text{Min } z^* = 5y_1 + 2y_2' - 2y_2''$$

S.t

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$$y_1 + 2y_2' - 2y_2'' \geq 5$$
$$2y_1 - y_2' + y_2'' \geq 12$$
$$y_1 + 3y_2' - 3y_2'' \geq 4$$

$$y_1, y_2', y_2'' \geq 0.$$

$$\text{Min } z^* = 5y_1 + 2(y_2' - y_2'')$$

s.t.

$$y_1 + 2(y_2' - y_2'') \geq 5$$
$$2y_1 - (y_2' - y_2'') \geq 12$$
$$y_1 + 3(y_2' - y_2'') \geq 4$$

$$\text{Min } z^* = 5y_1 + 2y_2$$

s.t.

$$y_1 + 2y_2 \geq 5$$
$$2y_1 - y_2 \geq 12$$
$$y_1 + 3y_2 \geq 4$$

$y_1 \geq 0$ & y_2 is unrestricted in sign.

ues Write the dual for the LPP

$$\text{Max } z = 3x_1 + 5x_2 + 7x_3$$

st

$$x_1 + x_2 + 3x_3 \leq 10$$
$$4x_1 - x_2 + 2x_3 \geq 15$$

$x_1, x_2 \geq 0$ & x_3 is unrestricted

Since x_3 is unrestricted so it is replaced by $(x_3' - x_3'')$ where x_3' & x_3'' both are non - negative

Then the given problem becomes

$$\text{Max } z = 3x_1 + 5x_2 + 7(x_3' - x_3'')$$

s.t.

$$x_1 + x_2 + 3(x_3' - x_3'') \le 10$$

$$-4x_1 + x_2 - 2(x_3' - x_3'') \le -15$$

The dual of the problem is

$$\text{Min } z^* = 10y_1 - 15y_2$$

s.t.

$$y_1 - 4y_2 \ge 3$$

$$y_1 + y_2 \ge 5$$

$$3y_1 - 2y_2 \ge 7$$

$$-3y_1 + 2y_2 \ge -7$$

$$y_1, y_2 \ge 0.$$

$$\text{Min } z^* = 10y_1 - 15y_2$$

st

$$y_1 - 4y_2 \ge 3$$

$$y_1 + y_2 \ge 5$$

$$3y_1 - 2y_2 = 7$$

$$y_1, y_2 \ge 0$$

Note The two problems primal and the dual are related to each other in the following manner :

- If one is a maximization problem then the other is a minimization problem.
- If one of them has an optimum solution then the other also has an optimum solution.

The sol^n of the other problem can be read from the net evaluation $Z_j - C_j$ row below the column of slack variables.
Both will have the same optimum value i.e. Max Z = Min Z^*

- If the primal problem has an ~~advantage~~ unbounded sol^n then the dual has either no sol^n or an unbounded sol^n.

- If ith dual constraints is multiplied by -1 then the ith prime variable computed from $Z_j - C_j$ row of the dual problem must be multiplied by -1.

- If dual has no feasible sol^n then the primal also admit no feasible sol^n.

⇒ **Dual Simplex Method.**

$$\text{Max}\left\{ \frac{Z_j - C_j}{y_{ij}} \; ; \; y_{ij} < 0 \right\}$$