unbounded solution.

Date 15/02/23 — General Linear Programing Problem.

Let z be a linear function on $\mathbb{R}^n$ defined by $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ where c_j's are constants. Let (a_{ij}) be an $m \times n$ real matrix and let $\{b_1, b_2, \ldots, b_m\}$ be a set of constants such that

(b)
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq \text{ or } \leq \text{ or } = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geq \text{ or } \leq \text{ or } = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geq \text{ or } \leq \text{ or } = b_m \end{cases}$$

(c) — $x_j \geq 0 \quad j = 1, 2, \ldots n$

The problem of determining an n-tuple $(x_1, x_2, \ldots x_n)$ which makes z a minimum (or maximum) and which satisfies ⓑ & ⓒ is called the general LPP.

⇒ Canonical and Standard form of LPP.

The Canonical form

$$\text{Max } z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

S.t

$$a_{i1}x_1 + a_{i2}x_2 + \cdots a_{in}x_n \leq b_i$$

$$x_j \geq 0 \qquad i = 1, 2, \ldots m$$

$$j = 1, 2, \ldots n$$

By making use of some elementary transformations. This form of LPP is called the canonical form of LPP.
The characteristic of this form are:

(i) The objective function is of the maximization type.

$$\text{Min } z = c_1x_1 + c_2x_2 + c_3x_3$$

$$\text{Max} - z = -c_1x_1 - c_2x_2 - c_3x_3$$

$$\text{Max } z^* = -c_1x_1 - c_2x_2 - c_3x_3$$

(ii) All the constraints are of the ≤ type except for the non negative restrictions.

(iii) All the variables are non-negative.
If x_3 is unrestricted variable,

$$x_3 = x_3' - x_3''$$

$$x_3', x_3'' \geq 0$$

The Standard form

The general LPP is of the form Max or Min $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

s.t

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i$$

$$x_1, x_2, \dots, x_n \geq 0$$

is known as in standard form

The characteristic of this form are:

(i) All the constraints are expressed in the form of equations except for the non-negative restrictions.

(ii) The right hand side of each constraint equation is non-negative.

Ques Rewrite in standard form of the LPP

$$\text{Min } z = 2x_1 + x_2 + 4x_3$$

s.t

$$-2x_1 + 4x_2 \leq 4$$

$$x_1 + 2x_2 + x_3 \geq 5.$$

$$2x_1 + 3x_3 \leq 2$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

$x_3 = x_3' - x_3''$

$$\text{Max } z = -2x_1 - x_2 - 4(x_3' - x_3'') + 0x_4 + 0x_5 + 0x_6$$

$$-2x_1 + 4x_2 + x_4 = 4$$

$$x_1 + 2x_2 + (x_3' - x_3'') - x_5 = 5$$

$$2x_1 + 3(x_3' - x_3'') + x_6 = 2$$

$$x_1, x_2, x_3', x_3'', x_4, x_5, x_6 \geq 0$$