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16/02/23

Date _____
Page _____

Ques Reduce the following LPP to its standard form

$$\text{Max } z = x_1 - 3x_2$$

s.t.

$$-x_1 + 2x_2 \leq 15$$

$$x_1 + 3x_2 = 10$$

x_1, x_2 unrestricted in sign

$$-(x_1' - x_1'') + 2(x_2' - x_2'') + x_3 = 15$$

$$(x_1' - x_1'') + 3(x_2' - x_2'') = 0$$

$$x_1'', x_1', x_2', x_2'' \geq 0$$

⇒ Basic Solution

Given a system of m simultaneous linear equations in unknowns ($m \times n$)

$$Ax = b, \quad x^T \in \mathbb{R}^n$$

where A is an $m \times n$ matrix of rank m . Let B be any $m \times m$ submatrix formed by m linearly independent columns of A . Then a solution obtained by setting $n-m$ variables not associated with the columns of B equal to zero and solving the resulting system is called a basic solution to the given system of equations.

⇒ Degenerate Solution

A basic solution to the system is called degenerate if one or more of the basic variables vanish.

Ques Obtain all the basic solutions to the following system of linear equations

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$Ax = B$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

non basic variable

$$x_3 = 0$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$$

$$x_2 = 0$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}$$

$$x_1 = 0$$

$$Bx = b$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$-3x_1 = -6$$

$$x_1 = 2, x_2 = 1 \Rightarrow \text{Basic Variable}$$

$$x_3 = 0 \Rightarrow \text{Non-Basic Variable}$$

Similarly,

$$x_1 + x_3 = 4$$

$$2x_1 + 5x_3 = 5$$

$$3x_3 = -3$$

$$x_3 = -1, x_1 = 5 \Rightarrow \text{Basic variable}$$

$$x_2 = 0 \Rightarrow \text{Non-Basic variable}$$

$$2x_2 + x_3 = 4$$

$$x_2 + 5x_3 = 5$$

$$x_3 = 3, x_2 = 1/2 \Rightarrow \text{Basic variable}$$

$$x_1 = 0 \Rightarrow \text{Non-Basic variable}$$

We observe that all the three basic solutions are non-degenerate solⁿ.

ues Show that the following system of linear eqⁿs has a degenerate solution

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3$$

