

Ques- Using simplex method to solve

Max $Z = 4x_1 + 10x_2$

s.t $2x_1 + x_2 \leq 50$
 $2x_1 + 5x_2 \leq 100$
 $2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

$2x_1 + x_2 + x_3 = 50$
 $2x_1 + 5x_2 + x_4 = 100$
 $2x_1 + 3x_2 + x_5 = 90$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Max $Z = 4x_1 + 10x_2 + 0x_3 + 0x_4 + 0x_5$

Initial simplex table

| θ | C_B | x_B | y_1 | y_2 | y_3 | y_4 | y_5 | Min Ratio |
|----------------|-------|-------|-------|-------|-------|-------|-------|-----------|
| y_3 | 0 | 50 | 2 | 1 | 1 | 0 | 0 | 50 |
| y_4 | 0 | 100 | 2 | 5* | 0 | 1 | 0 | 20 → |
| y_5 | 0 | 90 | 2 | 3 | 0 | 0 | 1 | 30 |
| Net evaluation | | $Z=0$ | -4 | -10 | 0 | 0 | 0 | |

| θ | C_B | x_B | y_1 | y_2 | y_3 | y_4 | y_5 | MR |
|----------|-------|-------|---------------|-------|-------|---------------|-------|----|
| y_3 | 0 | 50 | 2 | 1 | 1 | 0 | 0 | 50 |
| y_2 | 10 | 20 | $\frac{1}{5}$ | 1 | 0 | $\frac{1}{5}$ | 0 | 20 |
| y_5 | 0 | 90 | 2 | 3 | 0 | 0 | 1 | 30 |

| | | | | | | | | |
|-----------------------|----|---------|-----------------|---|---|----------------|---|---------------|
| y_3 | 0 | 30 | $\frac{1}{5}$ * | 0 | 1 | $-\frac{1}{5}$ | 0 | 30 → |
| y_2 | 10 | 20 | $\frac{1}{5}$ | 1 | 0 | $\frac{1}{5}$ | 0 | |
| y_5 | 0 | 30 | $\frac{1}{5}$ | 0 | 0 | $\frac{2}{5}$ | 1 | |
| Net eval. $Z_j - C_j$ | | $Z=200$ | 0 | 0 | 0 | 2 | 0 | (No negative) |

$$x_2 = 20, x_1 = 0 \text{ (nonbasic)}$$

$Z = c_1 x_1$ Since net evaluation corresponding to nonbasic variable x_1 is zero therefore an alternative basic soln exist.

$$Z = c_2 x_2 = 10 \times 20$$

$$\boxed{\text{Max } Z = 200}$$

$$y_3 \left| \begin{array}{ccc|ccc} 0 & 150 & 1 & 0 & 5/8 & -1/8 & 0 \\ 10 & 25 & 2/5 & 1 & 0 & 1/5 & 0 \\ 0 & 30 & 1/5 & 0 & 0 & -3/5 & 0 \end{array} \right.$$

| B | C_B | x_B | y_1 | y_2 | y_3 | y_4 | y_5 | M.R | |
|-------|-------|-----------|-----------------------|-------|---------------|-------|--------------|-----|--|
| y_1 | 4 | 150 | 1 | 0 | 5/8 | -1/8 | 0 | | |
| y_2 | 10 | 25 | 0 | 1 | -1/4 | 1/8 | 0 | | |
| y_5 | 0 | 15 | 0 | 0 | -1/2 | -1/2 | 1 | | |
| | | $Z = 200$ | 0 | 0 | 0 | 2 | 0 | | |
| | | | <u>Net evaluation</u> | | | | | | |
| | | | | | $x_1 = 150/8$ | | $x_2 = 25/2$ | | |

Convex set - $x, y \in S$

$$\lambda x + (1-\lambda)y \in S \quad \forall \lambda \in [0, 1]$$

C. Combination -

$$\begin{bmatrix} 0 \\ 20 \\ 30 \\ 0 \\ 30 \end{bmatrix}, \begin{bmatrix} 250 \\ 25/2 \\ 0 \\ 0 \\ 15 \end{bmatrix}$$

$$x^* = \begin{bmatrix} (1-\lambda)150/8 \\ (25+15\lambda)/2 \\ 30\lambda \\ 0 \\ 15+15\lambda \end{bmatrix}$$

$\lambda \in [0, 1]$ infinitely many alternative soln.

If two basic (optimal) feasible solns are known, then the infinite number of basic (optimal) feasible solution can be obtained.