

Q.17

Ques → Find the minimum value of

$$Z = 10x_1 + 3x_2 + 4x_3 + 0x_4 + 0x_5 + 0x_6$$

s.t.

$$11x_1 + x_2 = 6x_3 + 3x_4 = 7$$

$$16x_1 + x_2 \leq 6x_3 \leq 5$$

$$3x_1 - x_2 - x_3 \leq 0$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$\text{Soln} \rightarrow \frac{11x_1 + x_2}{3} - \frac{2x_3 + x_4}{3} = \frac{7}{3}$$

$$16x_1 + x_2 - 6x_3 + x_4 = 5$$

$$3x_1 - x_2 - x_3 + x_4 = 0$$

$$x_i \geq 0$$

$$i = 1, \dots, 6$$

\* Initial simplex table :-

|       |                              | 16            | 10             | 1             | 2     | 0     | 0     | 0     | $\frac{RHS}{R_i}$ |
|-------|------------------------------|---------------|----------------|---------------|-------|-------|-------|-------|-------------------|
| B     | $\theta$                     | $x_6$         | $y_1$          | $y_2$         | $y_3$ | $y_4$ | $y_5$ | $y_6$ | min. ratio        |
| $y_4$ | 0                            | $\frac{7}{3}$ | $\frac{14}{3}$ | $\frac{2}{3}$ | -2    | 1     | 0     | 0     | $\frac{7}{2}$     |
| $y_5$ | 0                            | 5             | $\frac{16}{3}$ | 1             | -6    | 0     | 1     | 0     | $\frac{5}{1}$     |
| $y_6$ | 0                            | 0             | $\frac{3}{3}$  | -1            | -1    | 0     | 0     | 1     | 0                 |
|       | $\frac{10x_1 + 3x_2}{3} = 0$ | 0             | 0              | 1             | -2    | 0     | 0     | 0     |                   |

| B     | CB  | $x_B$ | $y_1$ | $y_2$    | $y_3$    | $y_4$ | $y_5$ | $y_6$   | Min Ratio |
|-------|-----|-------|-------|----------|----------|-------|-------|---------|-----------|
| $y_4$ | 0   | $7/3$ | 0     | $17/9$   | $-4/9$   | 1     | 0     | $-14/9$ |           |
| $y_5$ | 0   | 5     | 0     | $19/3$   | $-2/3$   | 0     | 1     | $-16/3$ |           |
| $y_6$ | 10Z | 0     | 1     | $-1/3$   | $-1/3$   | 0     | 0     | $1/3$   |           |
| $Z=0$ |     |       | 0     | $-110/3$ | $-115/3$ | 0     | 0     | $107/3$ |           |

each entry of  $\uparrow$

Since, the column vector  $y_3$  corresponding to net evaluation  $Z_j - C_j$  each negative. This indicates that there is an unbounded soln of the given LPP.

Ques  $\rightarrow$  Use Simplex Method to solve the LPP

$$\text{Max } Z = 3x_1 + 2x_2$$

s.t.

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Soln  $\rightarrow$  
$$\text{Max } Z = 3x_1 + 2x_2 + 0x_3 + 0x_4$$

$$x_1 + x_2 + x_3 = 4$$

$$x_1 - x_2 + x_4 = 2$$

| B              | CB | XB | y <sub>1</sub> | y <sub>2</sub> | y <sub>3</sub> | y <sub>4</sub> | Min ratio |
|----------------|----|----|----------------|----------------|----------------|----------------|-----------|
| y <sub>3</sub> | 0  | 4  | 1              | 1              | 1              | 0              | 4         |
| y <sub>4</sub> | 0  | 2  | 1              | -1             | 0              | 1              | 2 →       |
| Z = 0          |    |    | -3             | -2             | 0              | 0              |           |

| B              | CB | XB | y <sub>1</sub> | y <sub>2</sub> | y <sub>3</sub> | y <sub>4</sub> | Min ratio |
|----------------|----|----|----------------|----------------|----------------|----------------|-----------|
| y <sub>3</sub> | 0  | -2 | 0              | -2             | -1             | 1              | -2/0 →    |
| y <sub>1</sub> | 3  | 2  | 1              | -1             | 0              | 1              | 2/3 →     |
| Z = 6          |    |    | 0              | -3             | 0              | -3             |           |

$x_1 = 3, x_2 = 0$

Max Z = 9

| B              | CB | XB | y <sub>1</sub> | y <sub>2</sub> | y <sub>3</sub> | y <sub>4</sub> | Min ratio |
|----------------|----|----|----------------|----------------|----------------|----------------|-----------|
| y <sub>2</sub> | 1  | 1  | 0              | 1              | 1/2            | -1/2           |           |
| y <sub>1</sub> | 3  | 3  | 1              | 0              | 1/2            | 1/2            |           |
| Z = 12         |    |    | 0              | 0              | 3/2            | 1/2            |           |

$x_1 = 3, x_2 = 1$

Max Z =  $3x_1 + 2x_2$   
 $= 3 \times 3 + 2 \times 1 = 9 + 2 = 11$