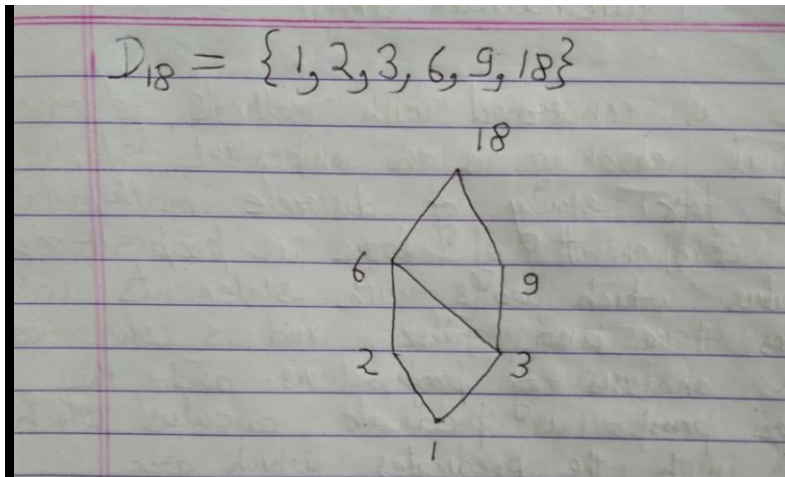

Subject Name: Mathematic II
Topic: Partial Order Relations and Lattices Part -II

By

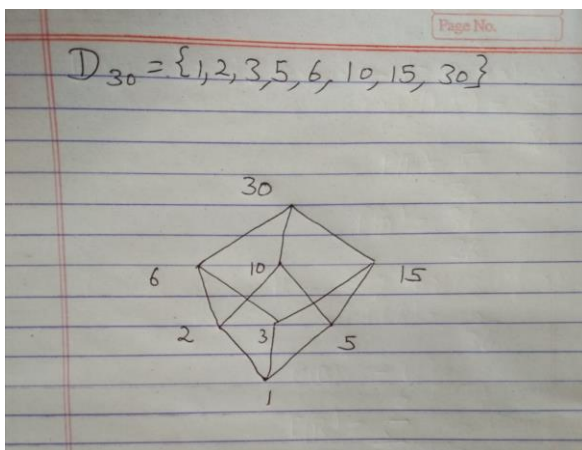
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Department of Computer Application
UIET, CSJM University, Kanpur

Hasse Diagram A partial order relation (\leq) on a finite set P can be represented by a diagram called the Hasse diagram.

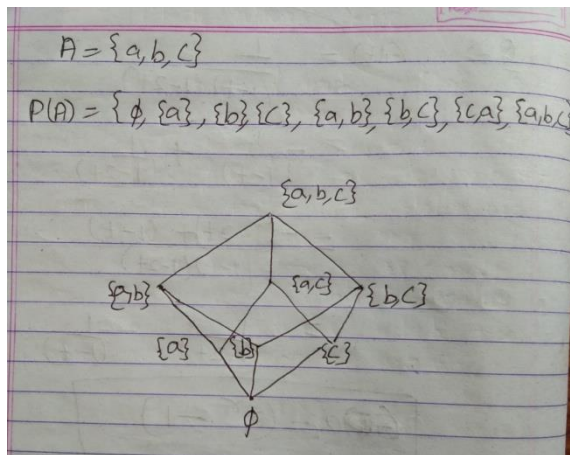
Ex. The Set D_{18} of positive integers, the relation “|” (divides). Its hasse diagram is



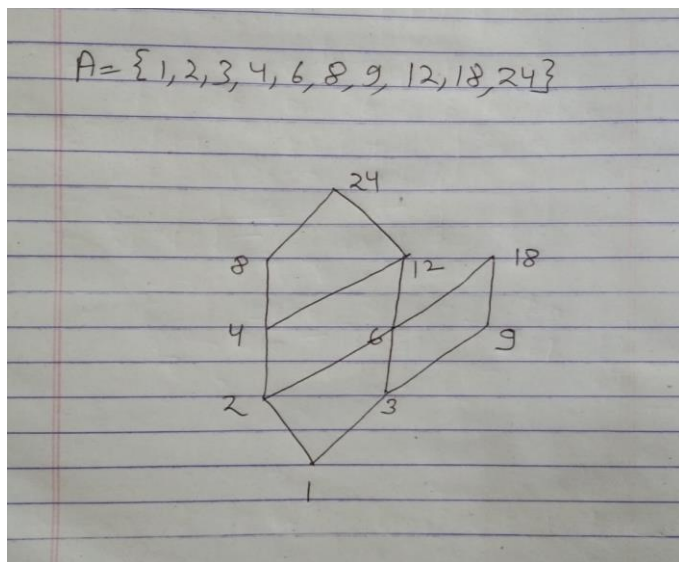
Q. the Hasse diagram of the set of positive integers divisible by 30 with the relation “|” is shown by



Q. Let $A = \{a, b, c\}$ and $P(A)$ its power set. Draw Hasse diagram of $(P(A), \subseteq)$.



Q. Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation “a divides b”. The Hasse diagram is



Maximal Element, Minimal Element

An element belonging to a point (a, \leq) is said to be Maximal element of A , if there is no element c in A such that $a < c$. An element $b \in A$ is said to be Minimal element of A if there is no element c in A such that $c < a$.

Date / /
Page No.

Q \Rightarrow Find all the maximal and minimal elements of posets whose Hasse diagrams are given in the diagrams

(i)

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graph BT; a((a)) --> b((b)); a --> f((f)); b --> c((c)); f --> d((d)); c --> e((e)); d --> e
```

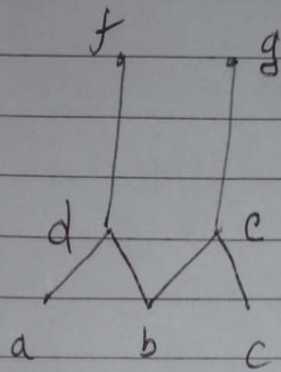
Maximal elements : c, e
Minimal elements : a, f

(ii)

```
graph BT; 1((1)) --> 2((2)); 2 --> 3((3)); 3 --> 4((4)); 3 --> 5((5))
```

Maximal elements = $4, 5$
Minimal elements = 1

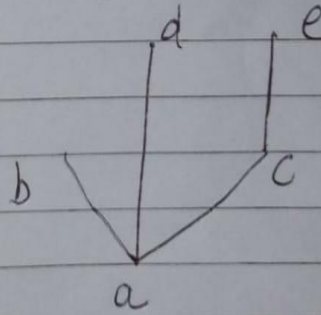
(III)



Maximal elements = f, g

Minimal elements = a, b, c

(IV)



Maximal elements = b, d, e

Minimal elements = a

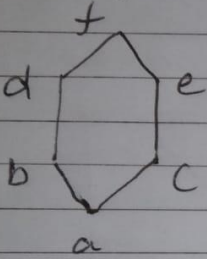
Greatest Element, Least Element

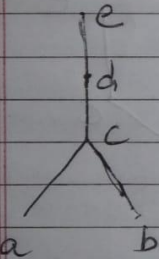
An element $a \in A$ is said to be a Greatest element of A , if $x \leq a$ for all $x \in A$. An element $a \in A$ is called a Least element of A , if $a \leq x$ for all $x \in A$.

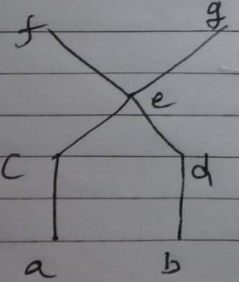
The least element is also called first element or zero element of A . the least element if exist is unique.

The greatest element is also called last element or unit element of A the greatest element if exist is unique.

Q find the greatest and least elements of the following Hasse diagrams!

(i)  Greatest element = f
Least element = a

(ii)  Greatest element = e
Least element = none

(iii)  Greatest element = none
Least element = none

Referential Books:

1. S.K. Sarkar, "Discrete Maths"; S. Chand & Co.,2000.
2. "Discrete Structure"; Shree Sai Prakashan.
3. "Discrete Mathematics", Schaum's Outlines.