PART- A

Co-ordinate System

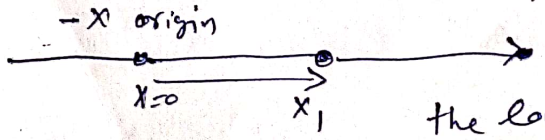
Position of any object in space describe by coordinate system. It is a artificial mathematic tool for describing the position of a real object. Co-ordinate system also help to understand the motion of particle or a system of particles. their are various Co-ordinate systems.

1. one dimensional coordinate systems (Number line)
2. Two dimensional cartesian coordinate systems
3. Polar Co-ordinate Systems
4. Three dimensional rectangular Cartesian system
5. Spherical ~~coordinates~~ polar coordinates.
6. cylindrical polar Co-ordinates.
7. Orthogonal Curvilinear coordinates.

1- One dimensional or Number line Co-ordinate System

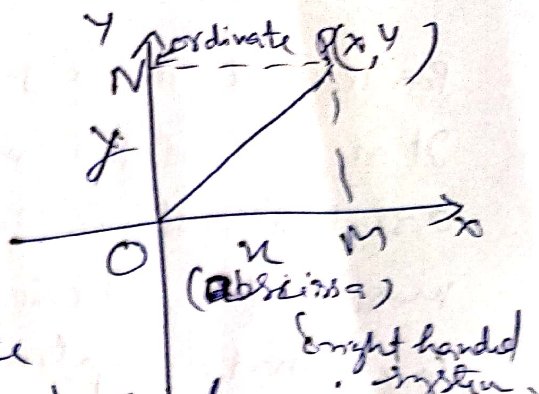
This Co-ordinate system describe the location of particle in one dimensional space (1-D). In this system only one Co-ordinate required for the defining location of an particle or object.

Example ⇒ A Bus is at position $x=0$ (x is single real number) which we call origin. If Bus is constrained to move along a straight line (x-axis), then all Points of East side is ~~Positive and~~ of the origin correspond to positive values of X and that of west side Corresponds to the negative values of x.



Here X-axis is our Co-ordinate system in which we can represent the location of a body by drawing an arrow starting from the origin in the direction of increasing x.

Two dimensional (2-D) Cartesian Co-ordinate system.



If particle hance moves in a plane but not along a straight line, then 1-D Coordinate system is not appropriate to describe the position of the particle. In this case 2-D Cartesian Co-ordinate System is used. 2-D Cartesian Coordinate System is formed by two mutually perpendicular axes which are intersect to eachother at O point (origin).

The coordinate of position of particle at P point is (x, y). If we draw a perpendicular from point P on x axis the foot of perpendicular meets on x-axis at M. the distance of M from origin is called x-coordinate of the point (abscissa) and similarly the distance of foot of perpendicular dropped from P on Y-axis from origin is called Y-Coordinate at that point or (ordinate). i.e

$OM = x$ and $ON = y$ and P point denoted by $P(x, y)$.

If $\hat{i}$ and $\hat{j}$ are unit vector along x and y axis then position vector at P point is

$$\vec{OP} = x\hat{i} + y\hat{j}$$

distance between two point $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in Cartesian Plane is

$$P_1P_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

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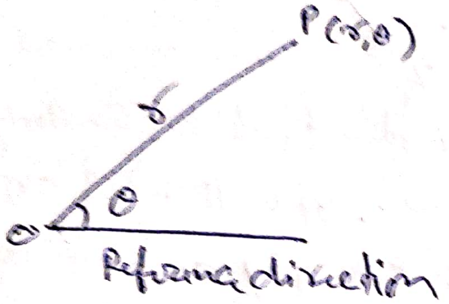
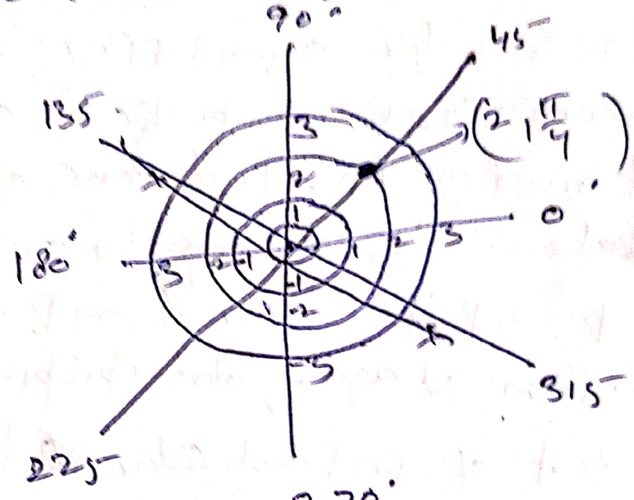
Polar Co-ordinates System

In this co-ordinate system every point on the plane is determined by a distance of the point from the reference origin (r) and angle (θ) from a reference direction instead of (x, y). Here r and θ are called Polar Coordinate and labeled as (r, θ)





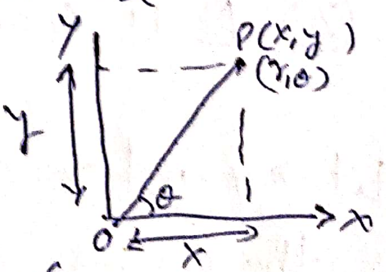
The distance of the point from origin is called radial co-ordinate or radius (r) and angle is called polar or azimuthal or angular co-ordinate.

A Positive θ measured Counter Clockwise from the axis. in this r co-ordinate is length of directed line from the pole and θ denotes the direction of r.

Range of r is $0 \to \infty$ and range of θ is 0 to π,

polar coordinate (r, θ) and Cartesian coordinate inter Converted

a).



$$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta \quad -(1)$$

$$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta \quad -(2)$$

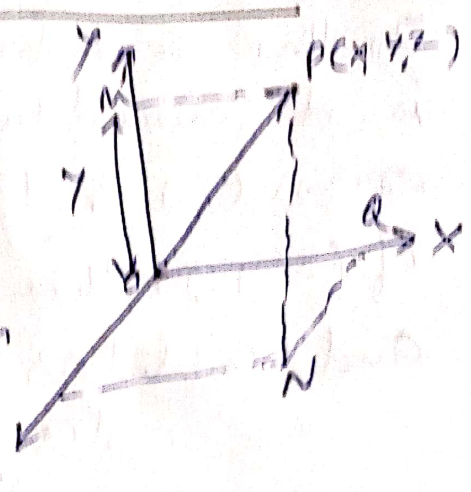
Also $x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta) = r^2 \Rightarrow r = \sqrt{x^2+y^2} \quad -(3)$

$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right) \quad -(4)$$

These any equation Convert polar coordinate (r, θ) to Cartesian coordinate (x, y) and vice versa.

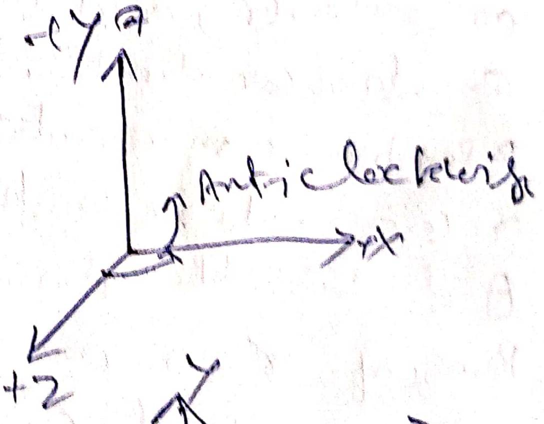
Three dimensional (3-D) Rectangular Cartesian Coordinate

This system based on three mutually Perpendicular Coordinate axes, (x, y and z). These axes intersect at a common point, called the origin O of the system.

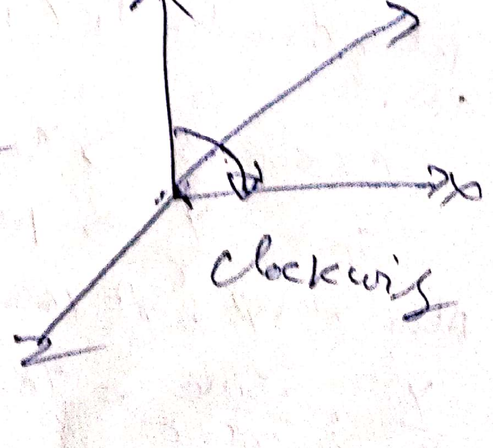


Now Consider a point P(x,y,z) some where in room. Draw a perpendicular on Z-axis. It meet on M. The distance of foot of Perpendicular M from the origin is Z-coordinate of P. Similarly drop a perpendicular from P on x-y plane. It will meet on N. from N again, draw perpendicular on x and y axis distance of foot of perpendicular from N on x and y axes i.e. P and Q from the origin O represents x and y coordinate at point P. The rectangular Cartesian system may be of two type

① R.H Coordinate System



② L.H Coordinate System



Let x, y and z axes are represented by three unit vectors $\hat{i}$ $\hat{j}$ and $\hat{k}$. Position vector of point P(x,y,z) can be written as

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$