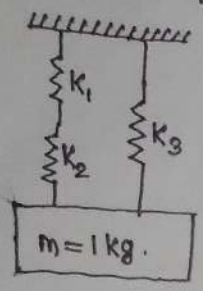
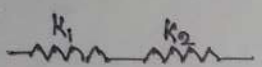
...ration.

Damped & undamped vibration

1.



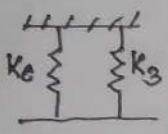
$K_1 = 1$ kN/m
$K_2 = 3$ kN/m
$K_3 = 2$ kN/m.



$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{K_e} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$$

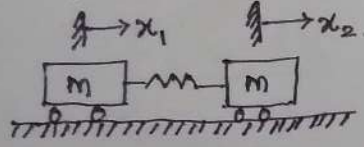
$$K_e = \frac{3}{4} = 0.75 \text{ KN/m}$$



$$K_e' = K_e + K_3 = 0.75 + 2$$
$$= 2.75 \text{ KN/m}$$

$$\omega_n = \sqrt{\frac{K_e'}{m}} = \sqrt{\frac{2750}{1}} = 52.44 \text{ rad/s}.$$

2.



$x_2 > x_1$ (Assume).

$m \rightarrow k(x_2 - x_1) \leftarrow m$

$$m\ddot{x}_1 = K(x_2 - x_1).$$

$$m\ddot{x}_1 + Kx_1 - Kx_2 = 0$$

$$m\ddot{x}_2 = -K(x_2 - x_1)$$

$$m\ddot{x}_2 - Kx_1 + Kx_2 = 0$$

$$M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \quad K = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}$$

$$K - M\omega^2$$

$$\begin{bmatrix} K - m\omega^2 & -K \\ -K & K - m\omega^2 \end{bmatrix}$$

$$\begin{vmatrix} K - m\omega^2 & -K \\ -K & K - m\omega^2 \end{vmatrix} = 0$$

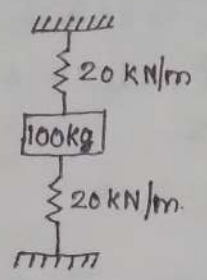
$$(K - m\omega^2)^2 - K^2 = 0$$

$$(K - m\omega^2)^2 = K^2.$$

$$K - m\omega^2 = K, \quad K - m\omega^2 = -K$$

$$\omega = 0, \quad \omega = \sqrt{\frac{2K}{m}}$$

3.



$K_{eq} = 20 + 20 = 40$ KN/m.

$m = 100$ kg.

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{40 \times 1000}{100}} = 20 \text{ rad/s}.$$

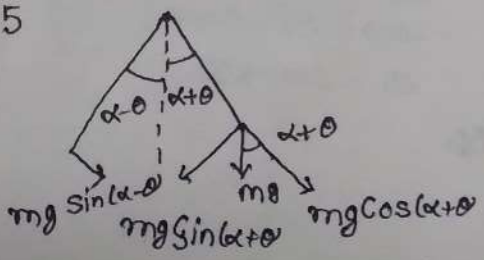
$$\omega_n = 2\pi f_n = 20$$

$$f_n = \frac{20}{2\pi} = \frac{10}{\pi}$$

4. $U_A =$

$$\Delta U = \frac{1}{2}kx^2 - mgx$$

5



$$-T = mgl\,\text{Sin}(\alpha+\theta) - mgl\,\text{Sin}(\alpha-\theta)$$

$$T = mgl[\text{Sin}(\alpha-\theta) - \text{Sin}(\alpha+\theta)]$$

$$I = 2ml^2$$

$$\alpha = \frac{mgl}{2ml^2}[\text{Sin}(\alpha-\theta) - \text{Sin}(\alpha+\theta)]$$

$$\alpha = -\frac{g}{2l}[\text{Sin}(\alpha+\theta) - \text{Sin}(\alpha-\theta)]$$

$$\text{Sin}(\alpha+\theta) = \text{Sin}\alpha\,\text{Cos}\theta + \text{Cos}\alpha\,\text{Sin}\theta$$

$$\text{Sin}(\alpha-\theta) = \text{Sin}\alpha\,\text{Cos}\theta - \text{Cos}\alpha\,\text{Sin}\theta$$

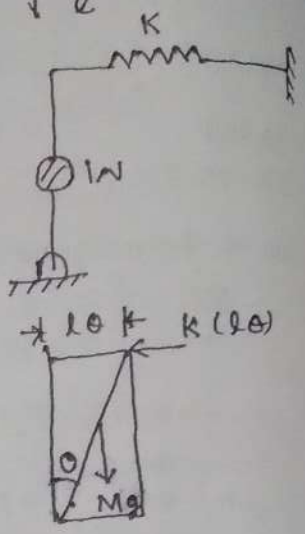
$$= 2\text{Cos}\alpha\,\text{Sin}\theta$$

$$\alpha = -\frac{g}{l}\text{Cos}\alpha\,\text{Sin}\theta$$

$$\omega^2 = \frac{g}{l}\text{Cos}\alpha$$

$$\omega = \sqrt{\frac{g\text{Cos}\alpha}{l}}.$$

6.



$$Kl\theta \times l\,\text{Cos}\theta = Mg\frac{l}{2}\text{Sin}\theta$$

$$Kl^2\theta\,\text{Cos}\theta = \frac{mgl}{2}\text{Sin}\theta$$

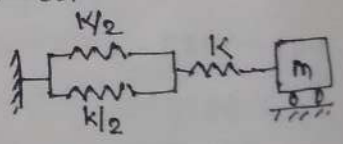
$$K = \frac{Wl}{2l^2}\frac{\text{Sin}\theta}{\theta} \times \frac{1}{\text{Cos}\theta}$$

$$K = \frac{W}{2l} = \frac{3000}{2 \times 0.3}$$

$$= 500$$

7. – d.

8.



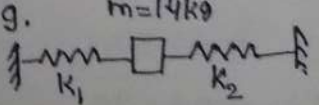
$$K_e = \frac{K}{2} + \frac{K}{2} = K.$$

$$\frac{1}{K_e'} = \frac{1}{K} + \frac{1}{K} = \frac{2}{K}$$

$$K_e' = \frac{K}{2}$$

$$\omega_n = \sqrt{\frac{K}{2m}}.$$

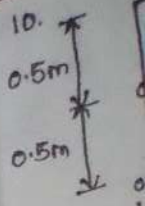
9.



$$K_e = K_1 + K_2$$
$$= 4000 + 1600$$
$$= 5600$$

$$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{5600}{1.4}} = 63.$$

$$f_n = 10 \text{ Hz}$$



$I\ddot{\theta} =$

$\ddot{\theta} =$

$I =$

$\omega_n^2 =$

$\omega_n^2 =$

$\omega_n =$

11.

13.