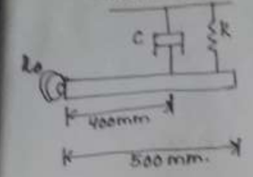
Forced Vibration.



$I\ddot{\theta} = -K_T\theta - KL^2\theta$

$\ddot{\theta} = -\frac{K_T + KL^2}{I}\theta$

$k_T = 1$ KN/m/rad.

$k = 2$ KN/m.

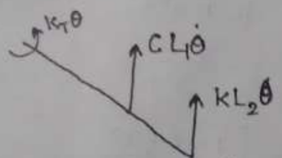
$L = 0.9$ m.

$I = \frac{mL^2}{12} + \frac{mL^2}{4} = \frac{mL^2}{3}$.

$\omega_n = \sqrt{\frac{K_T + KL^2}{I}} = \sqrt{\frac{1000 + 2000\times 0.9^2}{\frac{10}{3}(.9)^2}}$

$= \sqrt{\frac{2620}{2.7}} = 31.15$ rad/s.

27.



$I\ddot{\theta} + K_T\theta + kL_2^2\theta + cL_1^2\dot{\theta} = 0$

Damping co-efficient $= cL_1^2$

$= 500\times(0.4)^2 = 80$

28.

$C_c = 2m\omega_n$

$\omega_n = \sqrt{\frac{K}{M}}$

$\delta = \frac{PL^3}{3EI}$

$k = \frac{P}{\delta} = \frac{3EI}{L^3} = \frac{3\times 200\times I}{1^3}$

$I = \frac{bh^3}{12} = \frac{25^4}{12} = 32552\,mm^4$

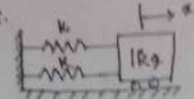
$k = 600\times 32552\times 10^{-12}\times 10^9$

$= 19531.2$ N/m

$\omega_n = \sqrt{\frac{19531.2}{20}} = 988.2$ rad/s.

$C_c = 2\times 20\times 988.2$

38.



$K_e = 2K = 40000$ N/m

$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{40000}{1}}$

$2\pi f = 200$ rad/s

$f = 31.8$ Hz

40 $\tan\phi = \frac{c\omega}{k - m\omega^2}$

At resonance.

$\omega = \omega_n = \sqrt{\frac{K}{m}}$.

$\therefore\ k - m\omega^2 = 0$

$\tan\phi = \infty$

$\phi = \frac{\pi}{2}$.

41 $\xi = \frac{C}{\sqrt{4km}} = \frac{5}{\sqrt{4\times 100\times 1}}$

$= \frac{5}{20} = \frac{1}{4} = 0.25$

42.

$X_0 = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$

$0.04 = \frac{10}{\sqrt{(6250 - 10\times 25^2)^2 + (c\omega)^2}}$

$0.04 = \frac{10}{c\omega} = \frac{10}{c\times 25}$

$c = 10$

43. $MF = \frac{X}{F/K} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$\omega = 1.5$ rad/s

$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{8}{2}} = 2$

$\frac{\omega}{\omega_n} = \frac{1.5}{2} = 0.75$

$MF = 2.28$

32 $C_c = \sqrt{4km}$

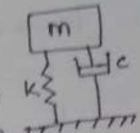
$= \sqrt{4\times 1000\times 12.5}$

$= 223.6$ Ns/m.

33 $\frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.42$.

$\xi = \frac{C}{C_c} = \frac{15}{223.6} = 0.067$

36.



$k = 3.6$ KN/m

$= 3600$ N/m

$C = 400$ Ns/m

$m = 50$ kg.

$\xi = \frac{C}{2\sqrt{km}} = \frac{400}{2\sqrt{3600\times 50}}$

$\xi = 0.471$.

$\omega_d = \omega_n\sqrt{1-\xi^2}$

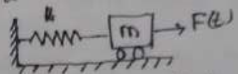
$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3600}{50}} = 8.48$

$\omega_d = 7.48$ rad/s.

$2\pi f_d = 7.48$ rad/s

$f_d = 1.19$ Hz.

37.



steady state amplitude

$X_0 = \frac{F_0}{|k - m\omega^2|}$

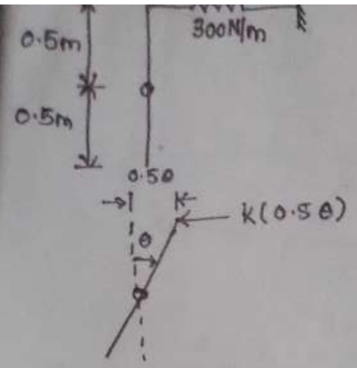
$= \frac{100}{|3000 - m\times 100^2|} = 0.05$

$\frac{100}{3000 - 10000m} = 0.05$

$100 = 150 - 500m$

$500m = 50$

$m = \frac{1}{10}$ kg.



$$I\ddot{\theta} = -\frac{k}{2}\theta \times \frac{1}{2}$$

$$\ddot{\theta} = -\frac{k}{2I}\theta$$

$$I = \frac{mL^2}{12} = \frac{1\times1^2}{12}$$

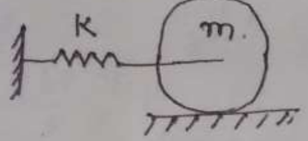
$$\omega_n^2 = \frac{k}{2I} = \frac{300}{2\times\frac{1}{12}}$$

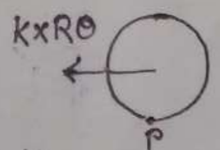
$$\omega_n^2 = 6\times300 = 3^2\times10^2\times2$$

$$\omega_n = 30\sqrt{2}\ rad/s.$$

$$= 42.4\ rad/s.$$

12.





$$I_P\ddot{\theta} = -kR\theta\cdot R$$

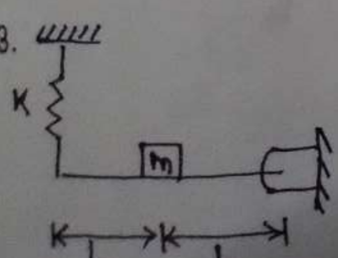
$$\ddot{\theta} = -\frac{kR^2}{I_P}\theta$$

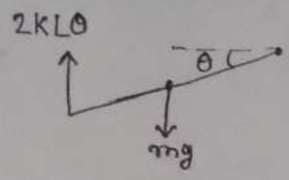
$$I_P = \frac{mR^2}{2} + mR^2 = \frac{3}{2}mR^2$$

$$\omega_n^2 = \frac{kR^2}{\frac{3}{2}MR^2} = \frac{2K}{3M}$$

$$\omega_n = \sqrt{\frac{2K}{3M}}.$$

13.





$$I\ddot{\theta} = -2KL\theta\times2L$$

$$\ddot{\theta} = -\frac{K4L^2}{mL^2}\theta$$

$$\omega = 2\sqrt{\frac{K}{m}}.$$

14. $a_{max} = \omega^2 A$

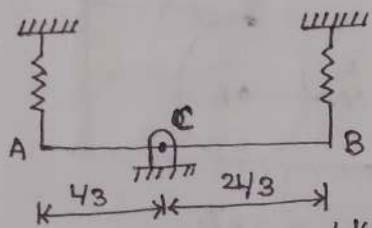
$f = 4Hz,\ A = 10mm = 0.01m$

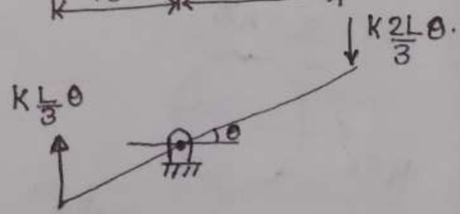
$2\pi f = 8\pi = 25.1\ rad/s.$

$a_{max} = (25.1)^2\times0.01.$

$= 6.3\ m/s^2.$

15.





$$I\ddot{\theta} = -K\frac{2L}{3}\theta\times\frac{2L}{3} - K\frac{L}{3}\theta\cdot\frac{L}{3}$$

$$= \left[-\frac{4KL^2}{9} - \frac{KL^2}{9}\right]\theta$$

$$= -\frac{5KL^2}{9}\theta$$

$$I = \frac{mL^2}{12} + m\left(\frac{L}{2}-\frac{L}{3}\right)^2$$

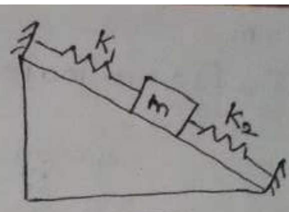
$$= \frac{mL^2}{12} + m\left(\frac{L}{6}\right)^2$$

$$= \frac{mL^2}{12} + \frac{mL^2}{36} = \frac{4}{36}mL^2 = \frac{mL^2}{9}$$

$$\omega_n^2 = \frac{5KL^2}{9\times\frac{mL^2}{9}} = \frac{5K}{m}$$

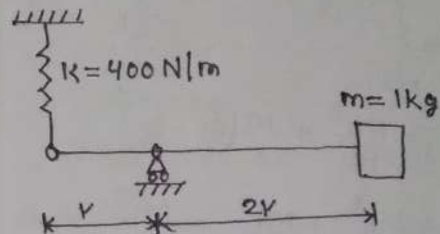
$$\omega_n = \sqrt{\frac{5K}{m}}.$$

16.



$$\omega_n = \sqrt{\frac{K_1+K_2}{m}}.$$

17



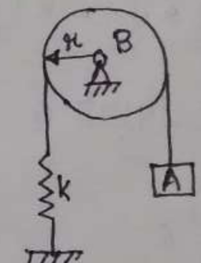
$$I = m\times(2r)^2 = 4r^2$$

$$I\ddot{\theta} = -Kr^2\theta$$

$$\ddot{\theta} = -\frac{Kr^2}{4r^2}\theta = -100\theta.$$

$$\omega = \sqrt{100} = 10$$

18.



$m_A = 5kg.$

$m_B = 20kg.$

$$E = \frac{1}{2}Kx^2 + \frac{1}{2}m_A\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2$$

$$x = R\theta.$$

$$E = \frac{1}{2}KR^2\theta^2 + \frac{1}{2}m_AR^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2$$

Given $K = 1500\ N/m$

$$\frac{dE}{dt} = KR^2\theta\dot{\theta} + \frac{m_AR^2}{2}2\dot{\theta}\ddot{\theta} + \frac{1}{2}I(2\dot{\theta})\ddot{\theta} = 0$$

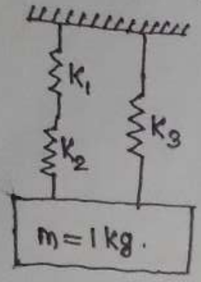
$$I\ddot{\theta} = -[KR^2 + m_AR^2]$$

$$[I + m_AR^2]\ddot{\theta} = -KR^2\theta$$

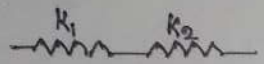
$$\omega^2 = \frac{KR^2}{I+m_AR^2} = \frac{1500R^2}{I+5R^2}$$

Damped & undamped Vibration

1.



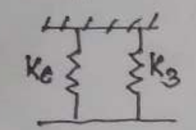
$K_1 = 1$ kN/m
$K_2 = 3$ kN/m
$K_3 = 2$ kN/m.



$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{K_e} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$$

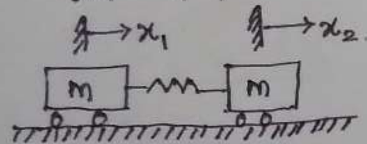
$$K_e = \frac{3}{4} = 0.75 \text{ KN/m}$$



$$K_e' = K_e + K_3 = 0.75 + 2$$
$$= 2.75 \text{ KN/m}$$

$$\omega_n = \sqrt{\frac{K_e'}{m}} = \sqrt{\frac{2750}{1}} = 52.44 \text{ rad/s.}$$

2.



$x_2 > x_1$ (Assume).

$\boxed{m} \rightarrow K(x_2 - x_1) \leftarrow \boxed{m}$

$$m\ddot{x}_1 = K(x_2 - x_1).$$

$$m\ddot{x}_1 + Kx_1 - Kx_2 = 0$$

$$m\ddot{x}_2 = -K(x_2 - x_1)$$

$$m\ddot{x}_2 - Kx_1 + Kx_2 = 0$$

$$M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \quad K = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}$$

$$K - m\omega^2$$

$$\begin{bmatrix} K - m\omega^2 & -K \\ -K & K - m\omega^2 \end{bmatrix}$$

$$\begin{vmatrix} K - m\omega^2 & -K \\ -K & K - m\omega^2 \end{vmatrix} = 0$$

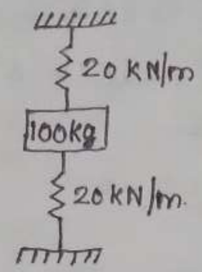
$$(K - m\omega^2)^2 - K^2 = 0$$

$$(K - m\omega^2)^2 = K^2.$$

$$K - m\omega^2 = K, \quad K - m\omega^2 = -K$$

$$\omega = 0, \quad \omega = \sqrt{\frac{2K}{m}}$$

3.



$$K_{eq} = 20 + 20 = 40 \text{ KN/m.}$$

$$m = 100 \text{ kg.}$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{40 \times 10\cancel{00}}{1\cancel{00}}} = 20 \text{ rad/s.}$$

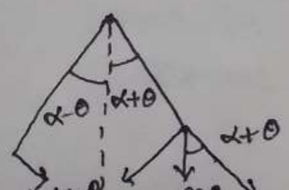
$$\omega_n = 2\pi f_n = 20$$

$$f_n = \frac{20}{2\pi} = \frac{10}{\pi}$$

4. $U_A =$

$$\Delta U = \frac{1}{2}Kx^2 - mgx$$

5



$$-T = mgl\sin(\alpha+\theta) - mgl\sin(\alpha-\theta)$$

$$T = mgl[\sin(\alpha-\theta) - \sin(\alpha+\theta)]$$

$$I = 2ml^2$$

$$\alpha = \frac{\cancel{m}gl}{2\cancel{m}\, l^2}[\sin(\alpha-\theta) - \sin(\alpha+\theta)]$$

$$\alpha = -\frac{g}{2l}[\sin(\alpha+\theta) - \sin(\alpha-\theta)]$$

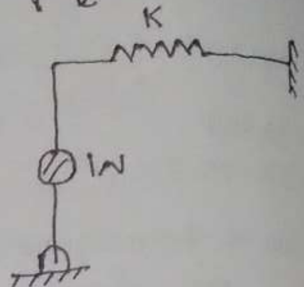
$$\sin(\alpha+\theta) = \sin\alpha\cos\theta + \cos\alpha\sin\theta$$
$$\sin(\alpha-\theta) = \sin\alpha\cos\theta - \cos\alpha\sin\theta$$
$$- \qquad\qquad - \qquad\qquad +$$
$$= 2\cos\alpha\sin\theta$$

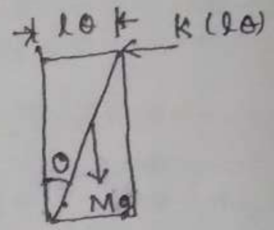
$$\alpha = -\frac{g}{l}\cos\alpha\sin\theta$$

$$\omega^2 = \frac{g}{l}\cos\alpha$$

$$\omega = \sqrt{\frac{g\cos\alpha}{l}}.$$

6.





$$Kl\theta \times l\cos\theta = Mg\frac{l}{2}\sin\theta$$

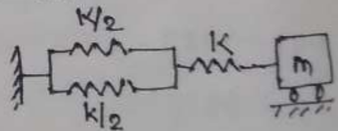
$$Kl^2\theta\cos\theta = \frac{mgl}{2}\sin\theta$$

$$K = \frac{Wl}{2l^2}\frac{\sin\theta}{\theta} \times \frac{1}{\cos\theta}$$

$$K = \frac{W}{2l} = \frac{3000}{2 \times 0.3}$$
$$= 500$$

7. – d.

8.



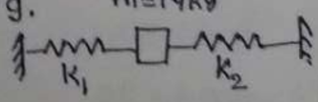
$$K_e = \frac{K}{2} + \frac{K}{2} = K.$$

$$\frac{1}{K_e'} = \frac{1}{K} + \frac{1}{K} = \frac{2}{K}$$

$$K_e' = \frac{K}{2}$$

$$\omega_n = \sqrt{\frac{K}{2m}}.$$

9. m = 14 kg

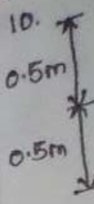


$$K_e = K_1 + K_2$$
$$= 4000 + 1600$$
$$= 5600$$

$$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{5600}{1.4}} = 63.$$

$$f_n = 10 \text{ Hz}$$

10.



$I\ddot{\theta} =$

$\ddot{\theta} =$

$I =$

$\omega_n^2 =$

$\omega_n^2 =$

$\omega_n =$

11.

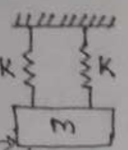
13.

19. $\Delta_s = 1\,mm$.

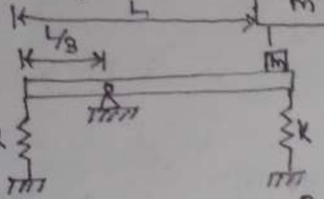
$$\omega_n = \sqrt{\frac{g}{\Delta_s}} = \sqrt{\frac{10}{0.001}} = 100\ rad/s.$$

20. $k_{eq} = 2K$.

$$\omega_n = \sqrt{\frac{2K}{m}}.$$



21.



$$I_1 = \frac{ML^2}{12} + M\left(\frac{L}{2} - \frac{L}{3}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{36} = \frac{ML^2}{9}$$

$$I_2 = m\left(\frac{2L}{3}\right)^2 = m \times \frac{4L^2}{9}$$

$$= \frac{M}{4} \times \frac{4L^2}{9} = \frac{ML^2}{9}.$$

$$I = I_1 + I_2 = \frac{2}{9}ML^2$$

$$I\ddot{\theta} = -K\frac{L}{3}\theta \cdot \frac{L}{3} - K\frac{2L}{3}\theta\frac{2L}{3}$$

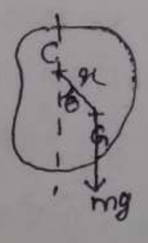
$$I\ddot{\theta} = -\frac{KL^2}{9} - \frac{4KL^2}{9}\theta$$

$$I\ddot{\theta} = -\frac{5KL^2}{9}\theta$$

$$\omega = \sqrt{\frac{5KL^2}{9 \times \frac{2}{9}ML^2}} = \sqrt{\frac{5K}{2M}}.$$

22. Radius of gyration = 100 mm.

$k = 100\,mm$.



$$I\ddot{\theta} = -mgr\theta$$

$$\ddot{\theta} = -\frac{mgr\theta}{mk^2}$$

$$\omega^2 = \sqrt{\frac{gr}{k^2}} = \sqrt{\frac{9.81 \times .25}{(0.1)^2}}$$

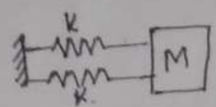
$= 15.7\ rad/s$.

23.

$$\frac{1}{K_e} = \frac{1}{K} + \frac{1}{K}$$

$$K_e = \frac{K}{2}$$

$$\omega_I = \sqrt{\frac{K_e}{M}} = \sqrt{\frac{K}{2M}}$$

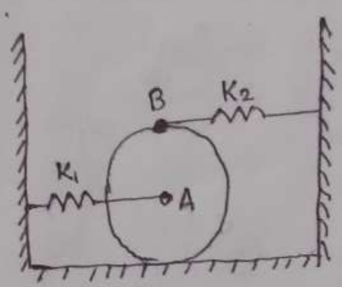


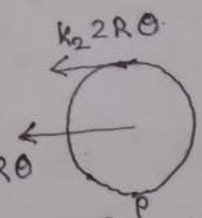
$$K_e = K + K = 2K.$$

$$\omega_{II} = \sqrt{\frac{K_e}{M}} = \sqrt{\frac{2K}{M}}.$$

$$\frac{\omega_I}{\omega_{II}} = \frac{1}{2}.$$

24.





$$I_p\ddot{\theta} = -K_1R^2\theta - K_2 4R^2\theta$$

$$I_p = \frac{mR^2}{2} + mR^2 = \frac{3}{2}mR^2$$

$$\omega = \sqrt{\frac{(K_1 + 4K_2)R^2}{\frac{3}{2}mR^2}} = \sqrt{\frac{K_1 + 4K_2}{\frac{3}{2}M}}.$$

$K_1 = 400\,N/m$

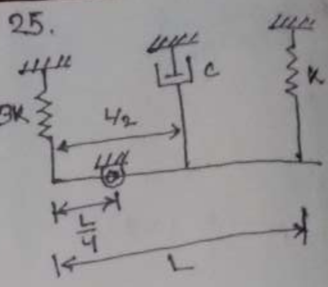
$K_2 = 100\,N/m$

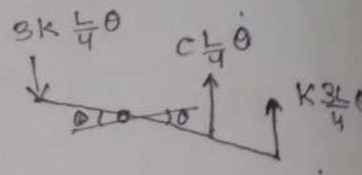
$M = 1\,kg$

$$\omega = \sqrt{\frac{800}{\frac{3}{2}}} = \sqrt{\frac{1600}{3}}$$

$= 23.09\ rad/s$.

25.





$$-I\ddot{\theta} = \frac{3KL}{4}\theta \times \frac{L}{4} + \frac{CL}{4}\dot{\theta}\frac{L}{4} + \frac{3KL\theta}{4}\frac{3L}{4}$$

$$+I\ddot{\theta} + \frac{3KL^2}{16}\theta + \frac{9KL^2}{16}\theta + \frac{CL^2}{16}\dot{\theta} = 0$$

$$I\ddot{\theta} + \frac{12KL^2}{16}\theta + \frac{CL^2}{16}\dot{\theta} = 0$$

$$I = \frac{mL^2}{12} + m\left(\frac{L}{4}\right)^2$$

$$= \frac{mL^2}{12} + \frac{mL^2}{16}$$

4 | 12, 16
3, 4

$$= \frac{(4+3)mL^2}{48}$$

$$= \frac{7mL^2}{48}$$

$$\frac{7mL^2}{48}\ddot{\theta} + \frac{12KL^2}{16}\theta + \frac{CL^2}{16}\dot{\theta} = 0$$

$$7m\ddot{\theta} + 12K\theta + C\dot{\theta} = 0$$

$$D = b^2 - 4ac = 0$$

$$\frac{7m}{3}s^2 + Cs + 12K = 0.$$

$$C^2 - 4 \times 7m \times 12K = 0$$

$$\frac{C}{\sqrt{Km}} = \sqrt{4 \times \frac{7}{3} \times 12} = 4\sqrt{7}$$