Assignment

Q.(1) Name the singularity in each case and give the region of convergence of each series.

(a) $\dfrac{e^{2z}}{(z-1)^3}$; $z=1$ (b) $(z-3)\sin\dfrac{1}{z+2}$; $z=-2$

(c) $\dfrac{z-\sin z}{z^3}$; $z=0$ (d) $\dfrac{z}{(z+1)(z+2)}$; $z=-2$

(e) $\dfrac{1}{z^2(z-3)^2}$; $z=3$

Q.(2). Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in a Laurent series valid for : (a) $1<|z|<3$, (b) $|z|>3$, (c) $0<|z+1|<2$

(d) $|z|<1$.

Q.(3) Calculate the residue of $f(z)=\dfrac{\cot z \coth z}{z^3}$ at $z=0$.

Q.(4). Evaluate $\dfrac{1}{2\pi i}\oint_C \dfrac{e^{zt}}{z^2(z^2+2z+2)}dz$ around the circle C with equation $|z|=3$.

Q.(5). Evaluate $\oint_C \dfrac{dz}{z-a}$ where C is any simple closed curve C and $z=a$ is (a) outside C, (b) inside C.

Q.(6). Using Cauchy's integral formula, evaluate the integral $\oint \dfrac{z^2}{z^2-1}dz$ around the unit circle with centre at (a) $z=1$, (b) $z=-1$, (c) $z=\dfrac{1}{2}$

Q.(7) Using Cauchy's integral formula calculate the integral $\int_c \frac{z\,dz}{(a-z^2)(z+i)}$ where c is the circle $|z|=2$ described in the positive sense.

Q.(8) Evaluate $\frac{1}{2\pi i}\oint_c \frac{e^z}{z-2}\,dz$ if C is (a) the circle $|z|=1$, (b) the circle $|z|=1$, using Cauchy's integral formula.