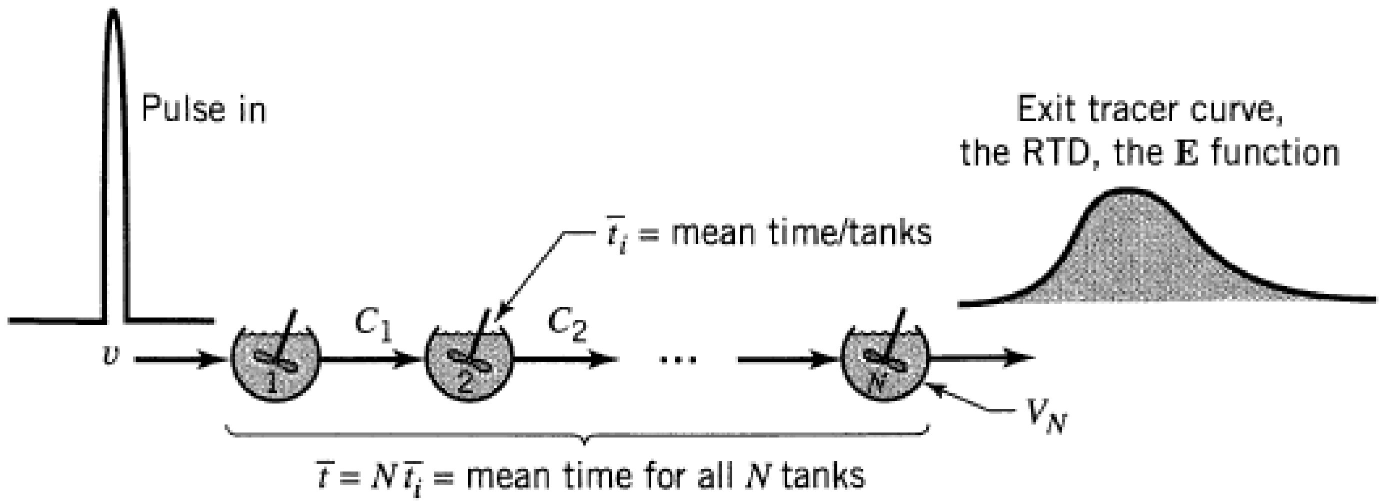
The Tanks-In-Series Model

Chapter 14

- This model can be used whenever the dispersion model is used and for not too large a deviation from plug flow both models give identical results for all practical purposes.
- The dispersion model has the advantage in that all correlations for flow in real reactors invariably use that model.
- On the other hand the tanks-in-series model is simple, can be used with any kinetics, and it can be extended without too much difficulty to any arrangement of compartments, with or without recycle.

PULSE RESPONSE EXPERIMENTS AND THE RTD



$\theta_i = \frac{t}{\bar{t}_i}$ = dimensionless time based on the mean residence time per tank $\bar{t}_i$

$\theta = \frac{t}{\bar{t}}$ = dimensionless time based on the mean residence time in all N tanks, $\bar{t}$.

$$\theta_i = N\theta \quad \ldots \quad \text{and} \quad \ldots \quad \bar{\theta}_i = 1, \quad \bar{\theta} = 1$$

$$\mathbf{E}_\theta = \bar{t}\mathbf{E}$$

For the first tank

- Consider a steady flow γ m^3/s of fluid into and out of the
- First of these ideal mixed flow units of volume V_1.
- At time $t = 0$ inject a pulse of tracer into the vessel which when evenly distributed in the vessel (and it is) has a concentration $\boldsymbol{C_0}$.
- At any time t after the tracer is introduced make a material balance, thus

$$\begin{pmatrix}\text{rate of disappearance}\\ \text{of tracer}\end{pmatrix} = \begin{pmatrix}\text{input}\\ \text{rate}\end{pmatrix} - \begin{pmatrix}\text{output}\\ \text{rate}\end{pmatrix}$$

$$V_1 \frac{dC_1}{dt} = 0 - vC_1 \qquad \left[\frac{\text{mol tracer}}{\text{s}}\right]$$

$$\int_{C_0}^{C_1} \frac{dC_1}{C_1} = -\frac{1}{\bar{t}_1}\int_0^t dt \quad \Longrightarrow \quad \frac{C_1}{C_0} = e^{-t/\bar{t}_1}$$

Since the area under this C/C_0 versus t curve is $\bar{t}_1$

- It allows you to find the E curve; so one may write

$$\bar{t}_1 \mathbf{E}_1 = e^{-t/\bar{t}_1}$$

For the second tank : where C_1 enters, C_2 leaves, a material balance gives

$$V_2 \frac{dC_2}{dt} = \upsilon \cdot \underbrace{\frac{C_0}{\bar{t}_1} e^{-t/\bar{t}_1}}_{C_1} - \upsilon C_2 \quad \left[\frac{\text{mol tracer}}{\text{s}}\right]$$

Separating gives a first-order differential equation, which when integrated gives

$$\bar{t}_2 \mathbf{E}_2 = \frac{t}{\bar{t}_2} e^{-t/\bar{t}_2}$$

The RTD's, means and variances, both in time and dimensionless time

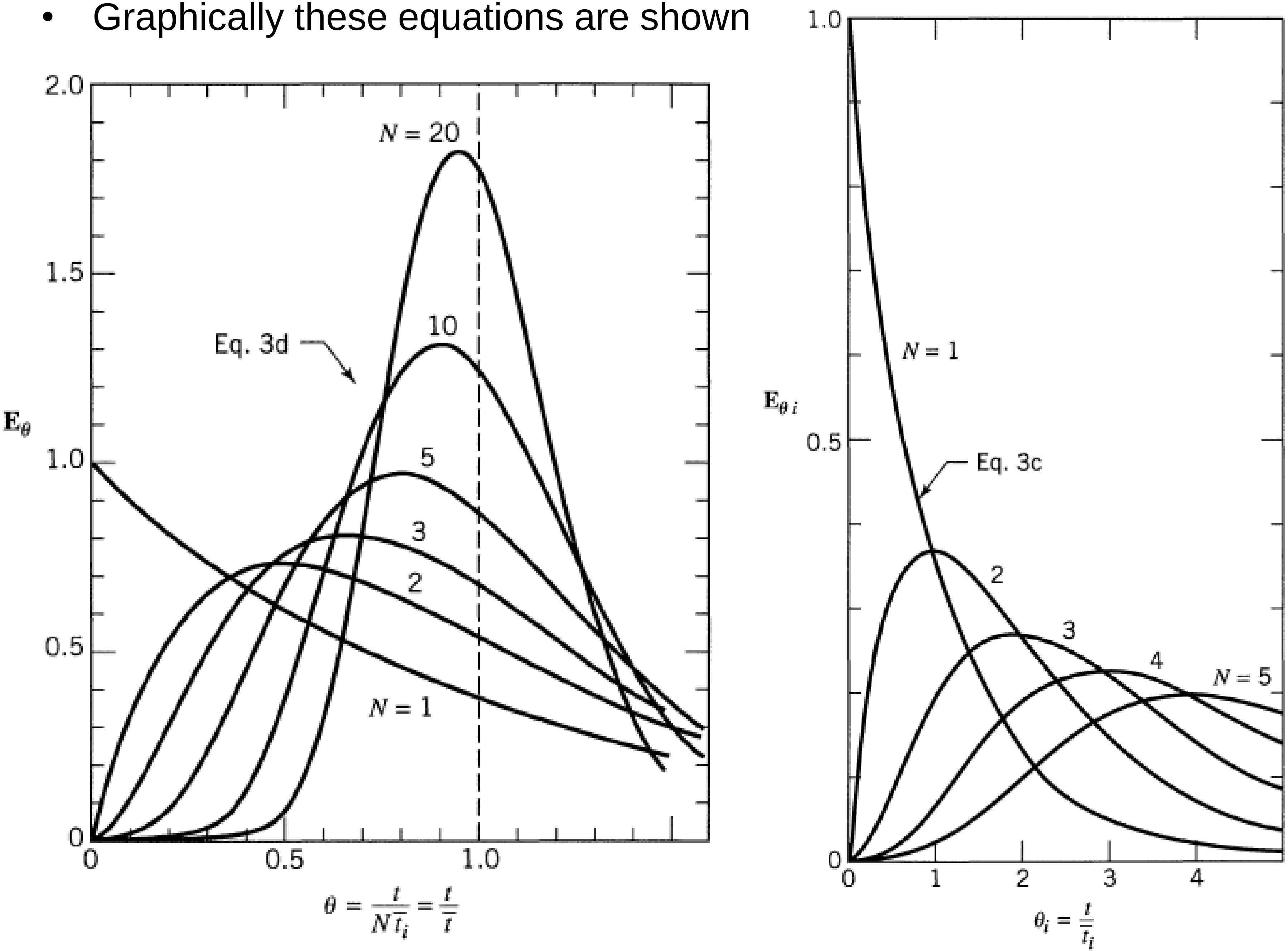
$$\bar{t}\, \mathbf{E} = \left(\frac{t}{\bar{t}}\right)^{N-1} \frac{N^N}{(N-1)!} e^{-tN/\bar{t}} \qquad \cdots \bar{t} = N\bar{t}_i \cdots \sigma^2 = \frac{\bar{t}^2}{N}$$

$$\bar{t}_i\, \mathbf{E} = \left(\frac{t}{\bar{t}_i}\right)^{N-1} \frac{1}{(N-1)!} e^{-t/\bar{t}_i} \qquad \cdots \bar{t}_i = \frac{\bar{t}}{N} \cdots \sigma^2 = N\bar{t}_i^2$$

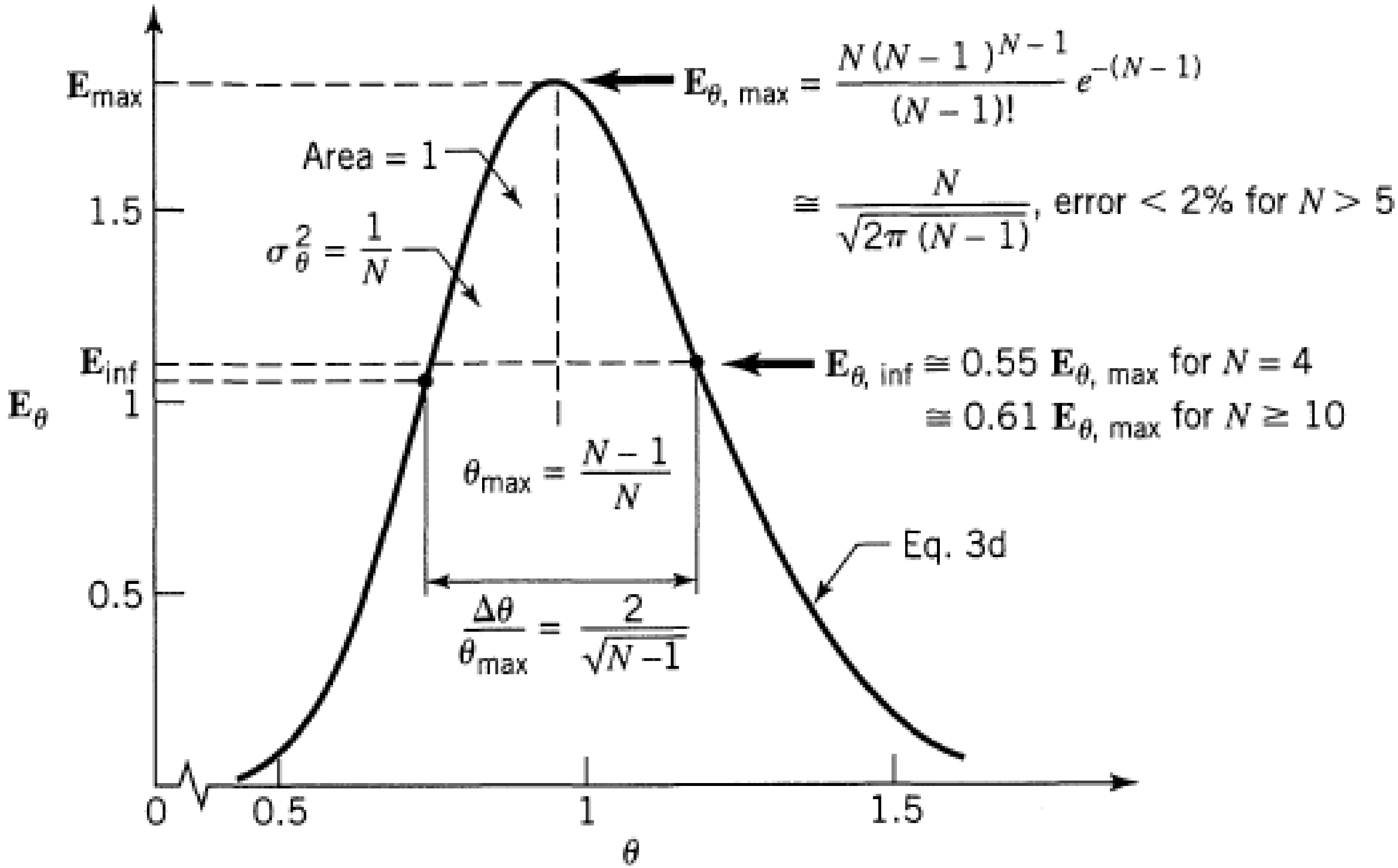
$$\mathbf{E}_{\theta i} = \bar{t}_i\, \mathbf{E} = \frac{\theta_i^{N-1}}{(N-1)!} e^{-\theta_i} \qquad \cdots \sigma_{\theta i}^2 = N$$

$$\mathbf{E}_\theta = (N\bar{t}_i)\, \mathbf{E} = N \frac{(N\theta)^{N-1}}{(N-1)!} e^{-N\theta} \cdots \sigma_\theta^2 = \frac{1}{N}$$

- Graphically these equations are shown



Properties of the RTD curve for the tanks-in-series model



Comments and Extensions

Independence. If M tanks are connected to N more tanks (all of the same size) then the individual means and variances (in ordinary time units) are additive

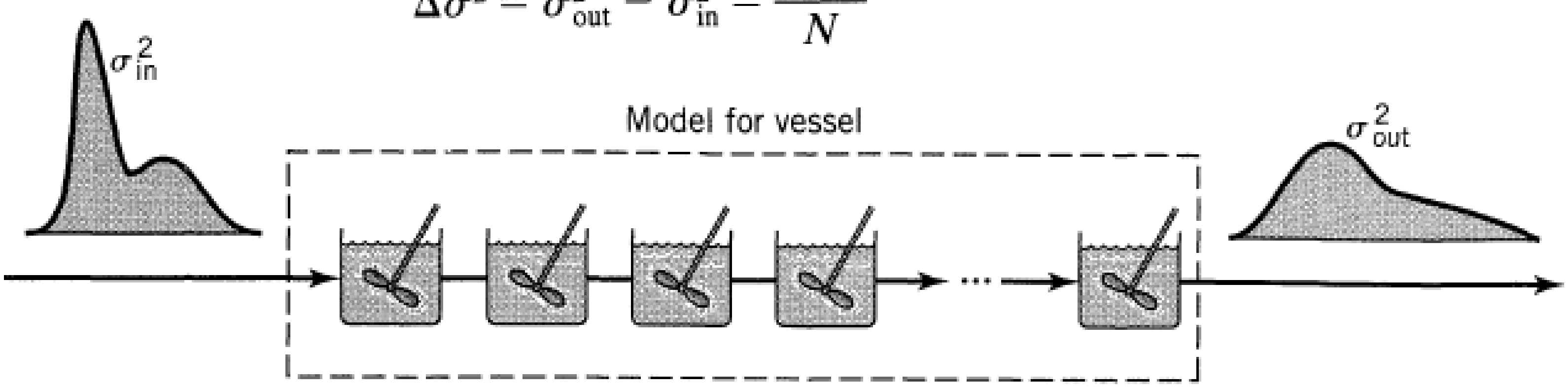
$$\bar{t}_{M+N} = \bar{t}_M + \bar{t}_N \ldots$$

$$\sigma^2_{M+N} = \sigma^2_M + \sigma^2_N$$

Because of this property we can join incoming streams with recycle streams. Thus this model becomes useful for treating recirculating systems

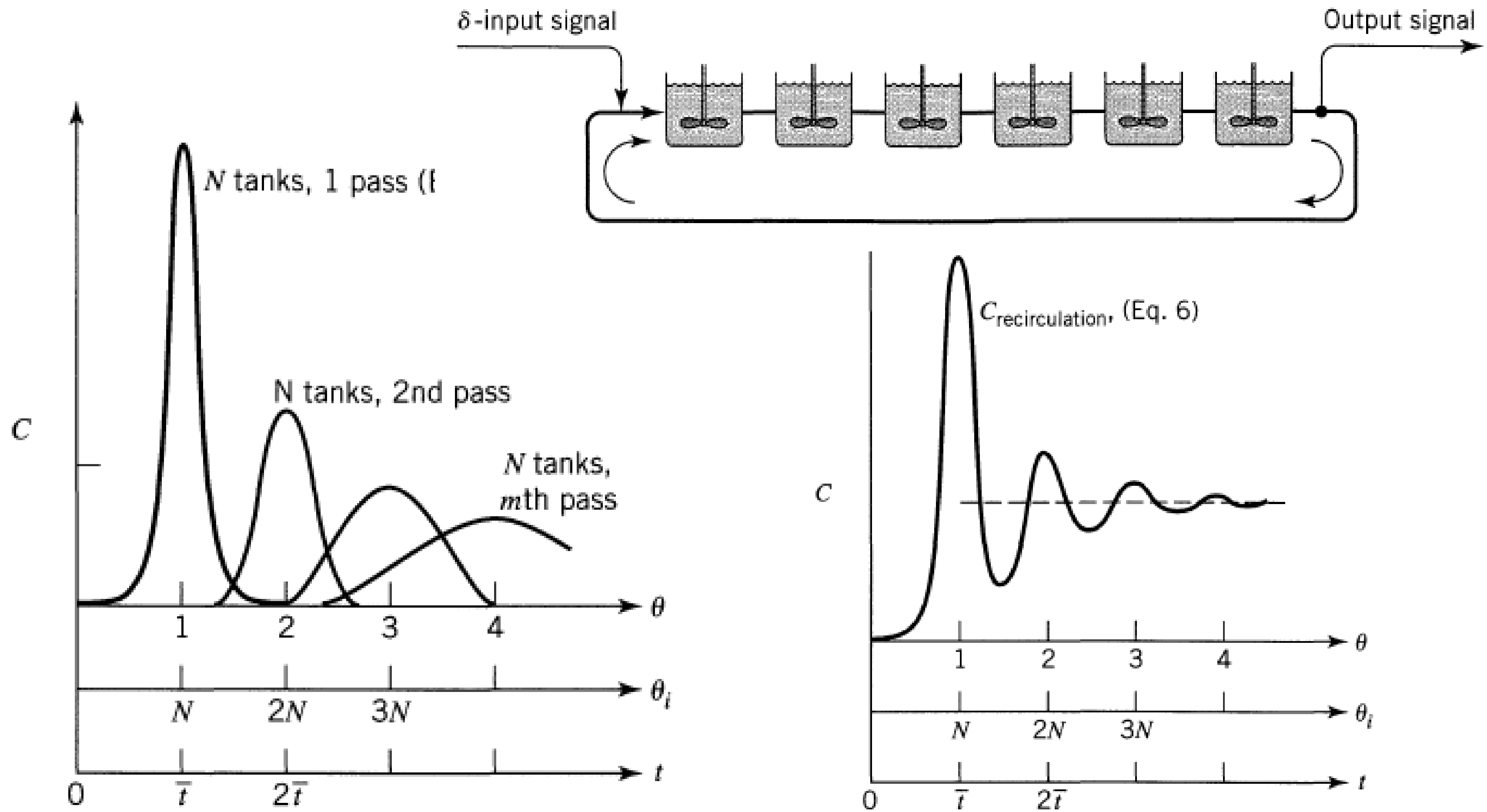
One-shot Tracer Input. If we introduce any one-shot tracer input into N tanks,

$$\Delta\sigma^2 = \sigma^2_{out} - \sigma^2_{in} = \frac{(\Delta\bar{t})^2}{N}$$



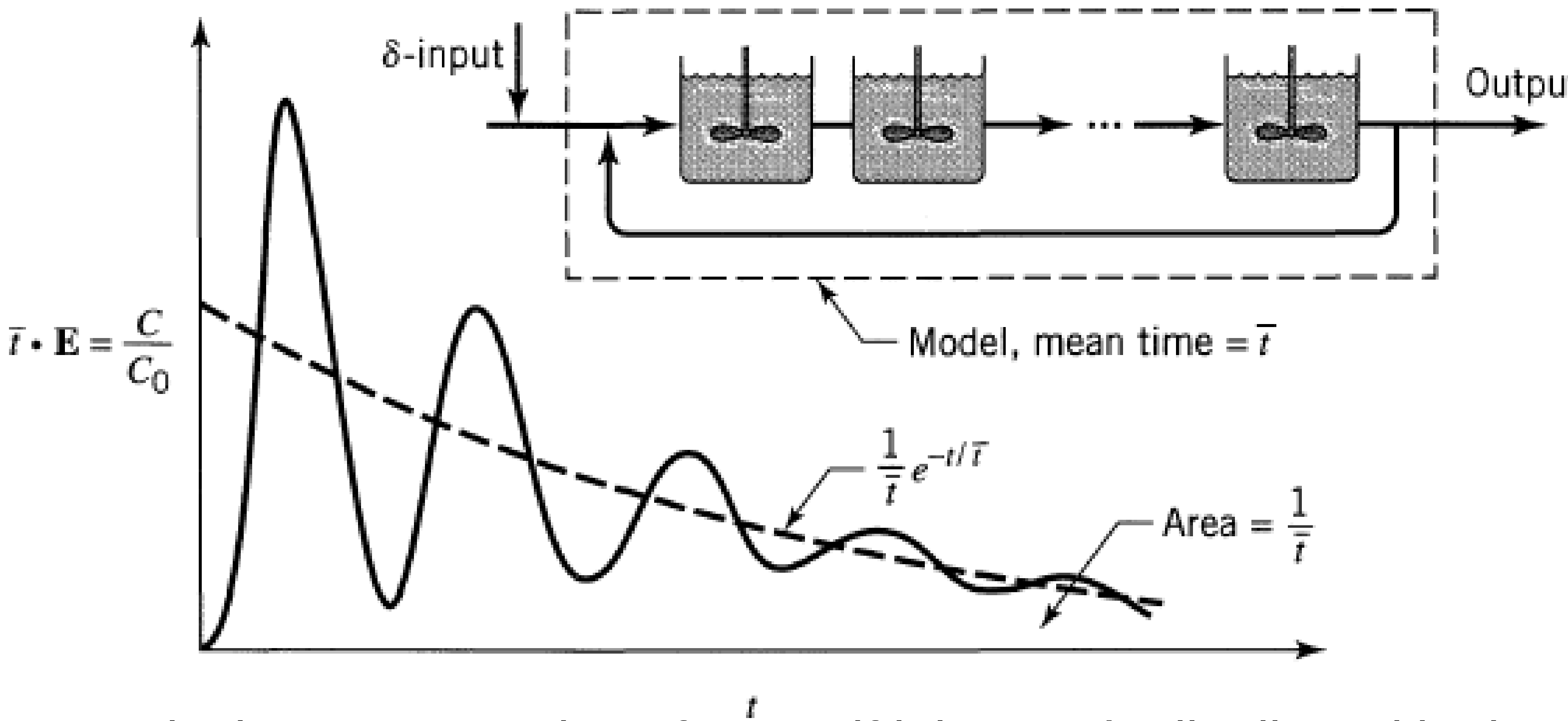
Closed Recirculation System

- If we introduce a δ signal into an N stage system,as shown in Fig. the recorder will measure tracer as it flows by the first time,
- the second time, and so on. In other words it measures tracer which has passed through N tanks, 2N tanks, and so on. In fact it measures the superposition of all these signals



Recirculation with Throughflow

- For relatively rapid recirculation compared to throughflow, the system as a whole acts as one large stirred tank;
- hence, the observed tracer signal is simply the superposition of the recirculation pattern and the exponential decay of an ideal stirred tank.



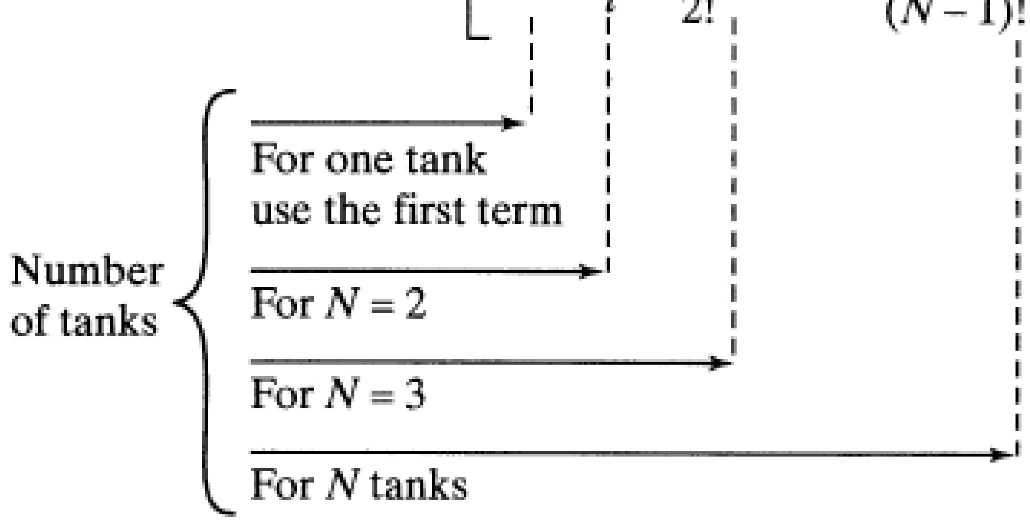
where ***Co*** is the concentration of tracer if it is evenly distributed in the system

- This form of curve is encountered in closed recirculation systems in which tracer is broken down and removed by a first-order process, or in systems using radioactive tracer.
- Drug injection on living organisms give this sort of superposition because the drug is constantly being eliminated by the organism.

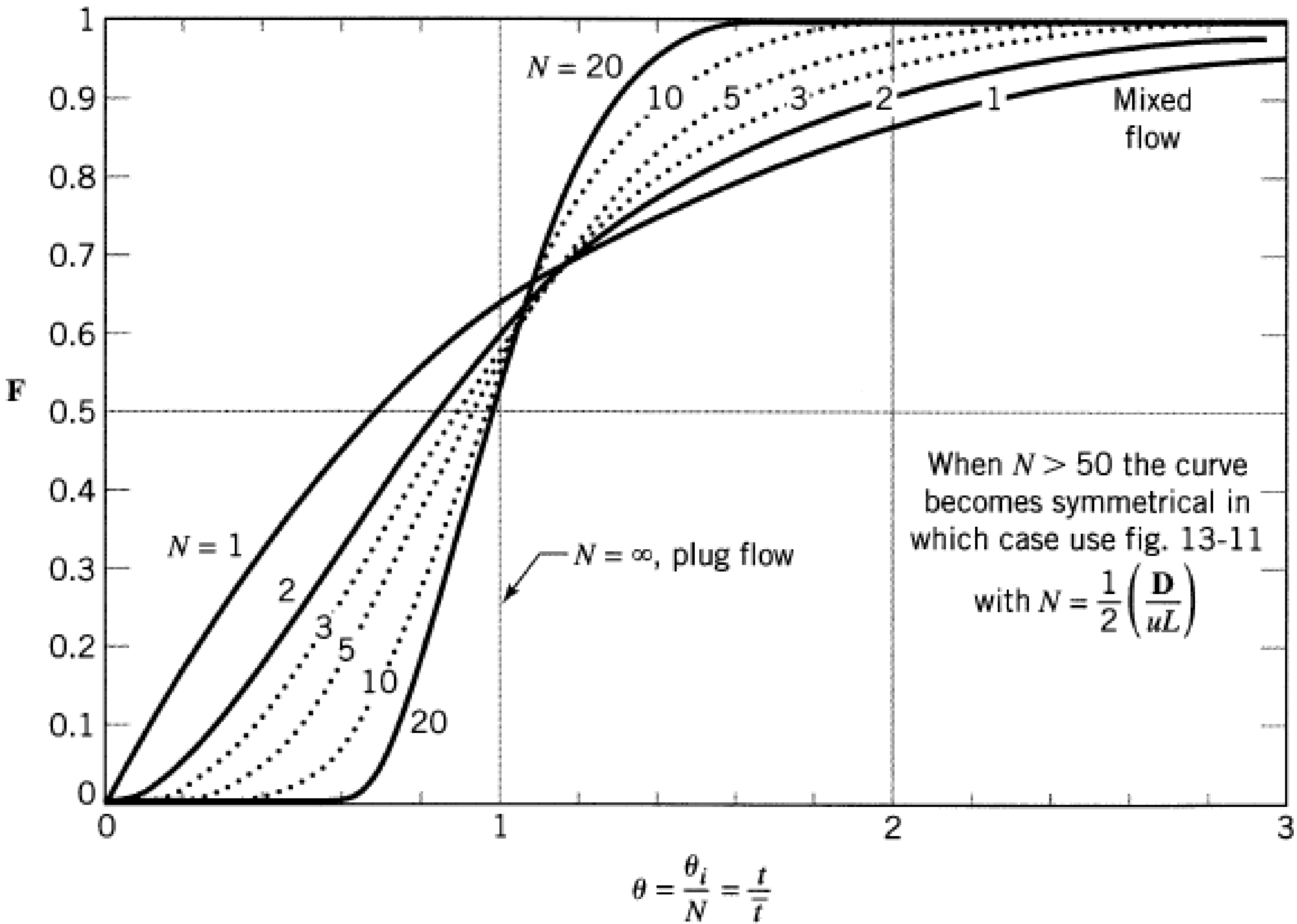
Step Response Experiments and the F Curve The output **F** curve from a series of N ideal stirred tanks is

$$\mathbf{F} = 1 - e^{-N\theta}\left[1 + N\theta + \frac{(N\theta)^2}{2!} + \cdots + \frac{(N\theta)^{N-1}}{(N-1)!} + \cdots\right]$$

$$\mathbf{F} = 1 - e^{-\theta_i}\left[1 + \theta_i + \frac{\theta_i^2}{2!} + \cdots + \frac{\theta_i^{N-1}}{(N-1)!} + \cdots\right]$$



The F curve for the tanks-in-series model



CHEMICAL CONVERSION

- Thus for first-order reactions in one tank

$$\frac{C_A}{C_{A0}} = \frac{1}{1 + k\bar{t}_i} = \frac{1}{1 + k\bar{t}}$$

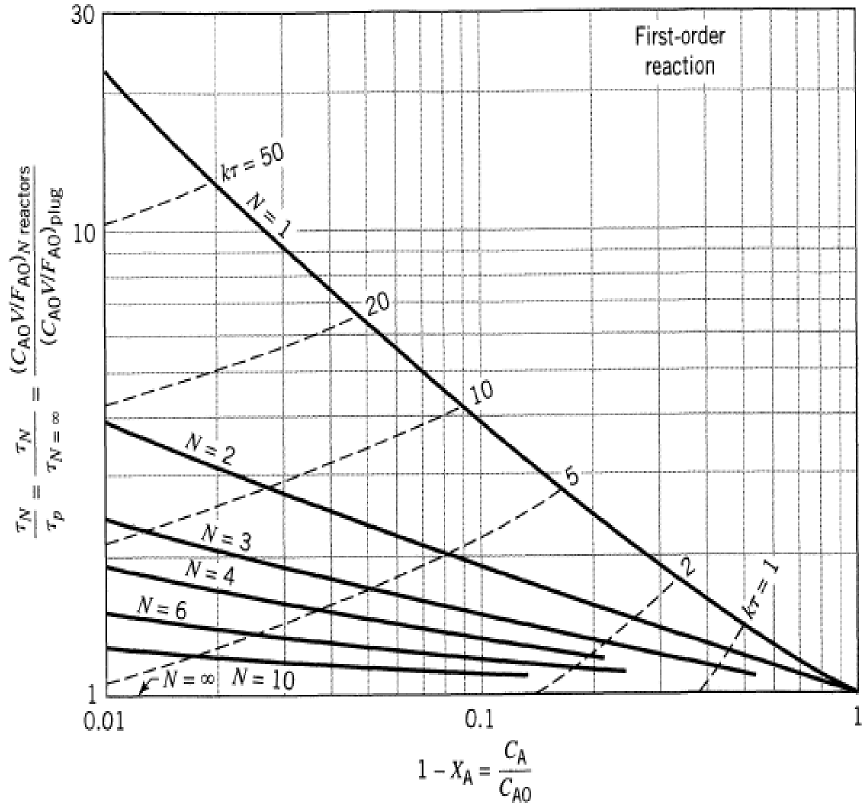
for N tanks in series

$$\frac{C_A}{C_{A0}} = \frac{1}{(1 + k\bar{t}_i)^N} = \frac{1}{\left(1 + \frac{k\bar{t}}{N}\right)^N}$$

(large ***N)*** comparison with plug flow gives

for same $C_{A\,\mathrm{final}}$: $$\frac{V_{N\,\mathrm{tanks}}}{V_p} = 1 + k\bar{t}_i = 1 + \frac{k\bar{t}}{2N}$$

for same volume V: $$\frac{C_{A,N\mathrm{tanks}}}{C_{Ap}} = 1 + \frac{(k\bar{t})^2}{2N}$$

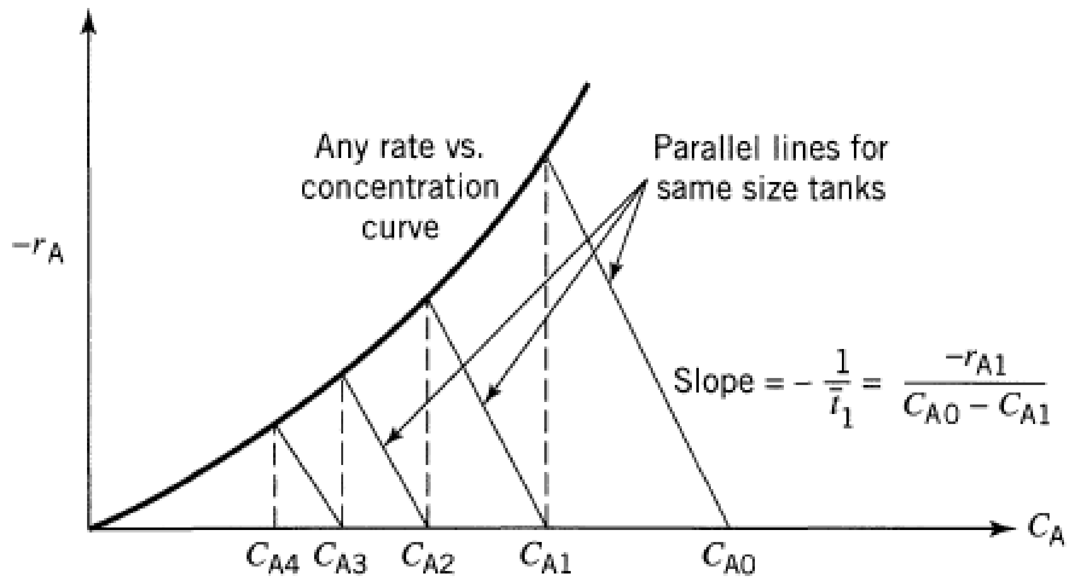


All Other Reaction Kinetics of Microfluids

- Either solve the mixed flow equation for tank after tank

$$\bar{t}_i = \frac{C_{Ai-1} - C_{Ai}}{-r_i}$$

Graphical method of evaluating the performance of N tanks in series for any kinetics.



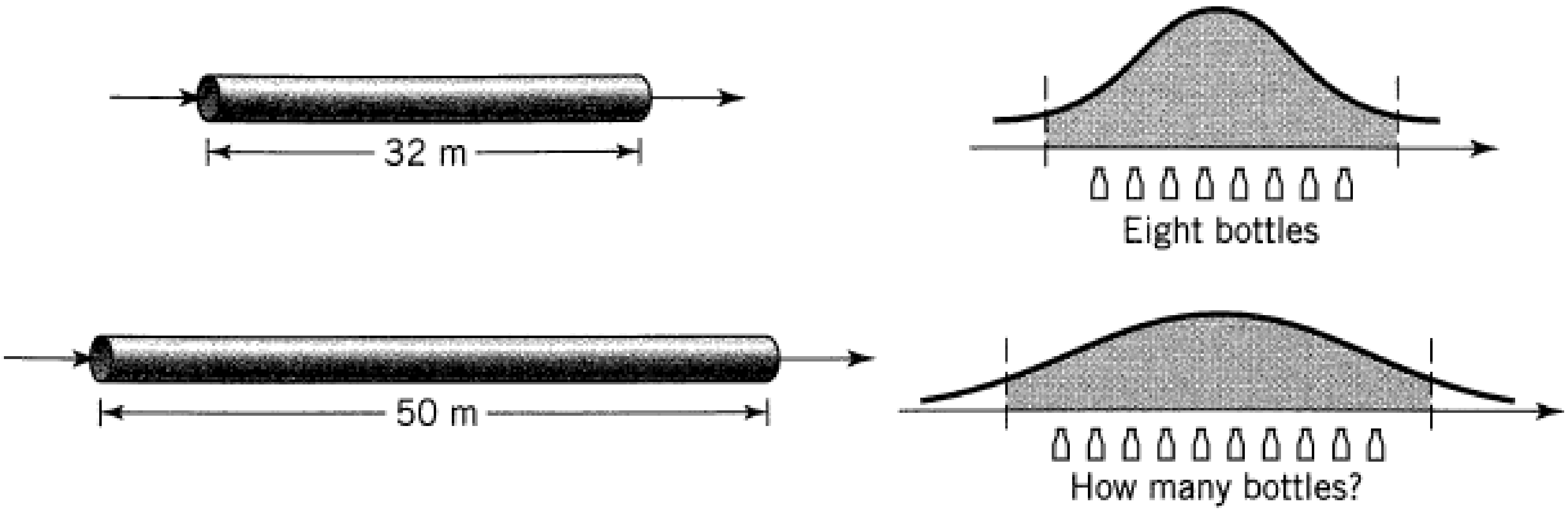
Chemical Conversion of Macrofluids

- There is rare use for macrofluid equations for homogeneous reactions
- There is rare use for macrofluid equations for homogeneous reactions

$$\frac{C_A}{C_{A0}} = \frac{N^N}{(N-1)! \cdot \bar{t}_N} \int_0^\infty \left(\frac{C_A}{C_{A0}}\right)_{\text{batch}} \cdot t^{N-1} e^{-tN/\bar{t}} \, dt$$

MODIFICATIONS TO A WINERY

A small diameter pipe 32 m long runs from the fermentation room of a winery to the bottle filling cellar. Sometimes red wine is pumped through the pipe, sometimes white, and whenever the switch is made from one to the other a small amount of "house blend" rosé is produced (8 bottles). Because of some construction in the winery the pipeline length will have to be increased to 50 m. For the same flow rate of wine, how many bottles of rosé may we now expect to get each time we switch the flow?



sketches the problem. Let the number of bottles, the spread, be related to *σ*.

Original:	$L_1 = 32\ \text{m}$	$\sigma_1 = 8$	$\sigma_1^2 = 64$
Longer pipe:	$L_2 = 50\ \text{m}$	$\sigma_2 = ?$	$\sigma_2^2 = ?$

But for small deviations from plug flow, from Eq

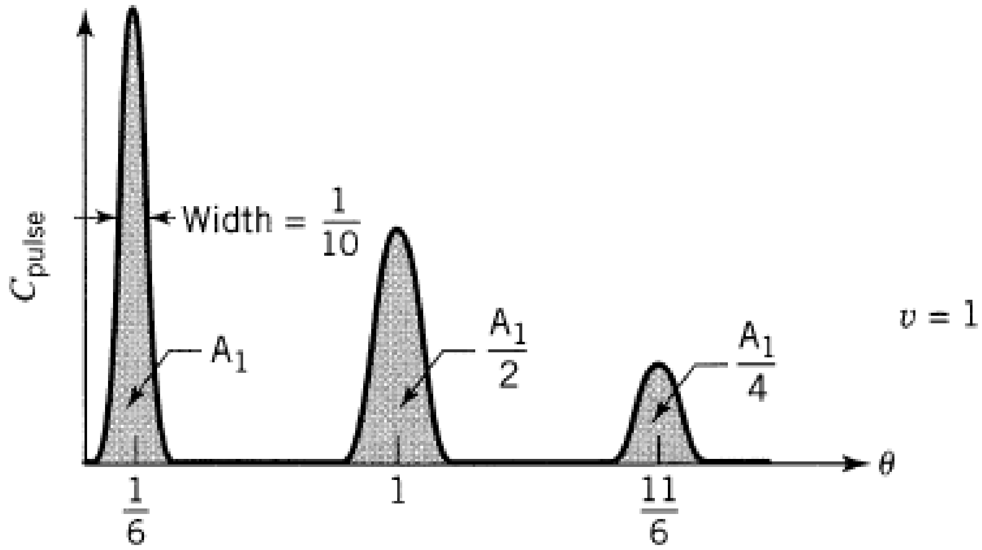
$$\sigma^2 \propto N \text{ or } \sigma^2 \propto \text{L}.$$

$$\therefore \frac{\sigma_2^2}{\sigma_1^2} = \frac{L_2}{L_1} = \frac{50}{32} \longrightarrow \therefore \sigma_2^2 = \frac{50}{32}(64) = 100$$

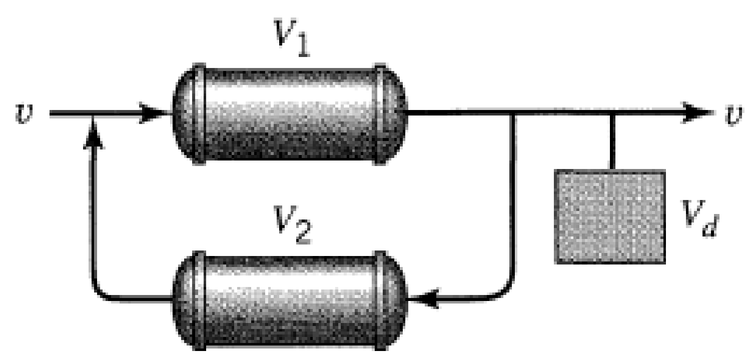
$\therefore \sigma_2 = 10$ . . . or we can expect <u>10 bottles of vin rosé</u>

FLOW MODELS FROM RTD CURVES

- Let us develop a tanks-in-series model to fit the RTD shown in Fig



- As a first approximation, assume that all the tracer curves are ideal pulses.
- We will later relax this assumption.
- Next notice that the first pulse appears early.
- This suggests a model as shown in Fig



- where v = 1 and $V_1 + V_2 + V_3 = 1$.
- In Chapter 12 we see the characteristics of this model, so let us fit it.
- Also it should be mentioned that we have a number of approaches. Here is one:
- Look at the ratio of areas of the first two peaks

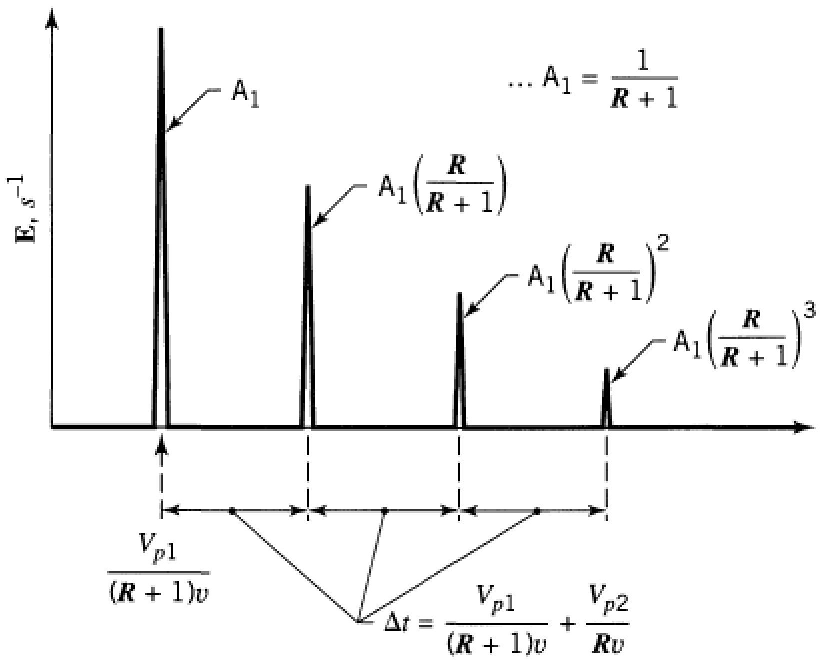
$$\frac{A_2}{A_1} = \frac{1}{2} = \frac{R}{R+1} \quad \cdots \underline{\underline{R = 1}}$$

- From the location of the first peak

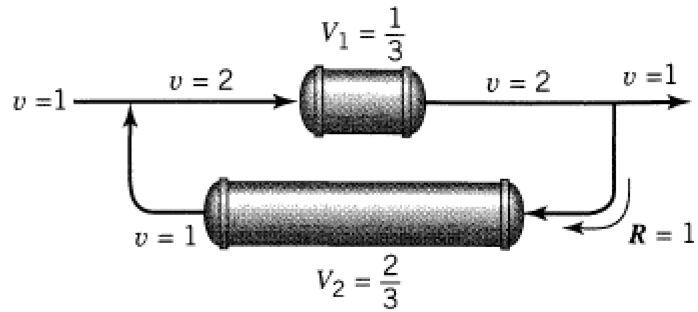
$$\frac{V_1}{(R+1)v} = \frac{V_1}{(1+1)} = \frac{1}{6} \quad \cdots \underline{\underline{V_1 = \frac{1}{3}}}$$

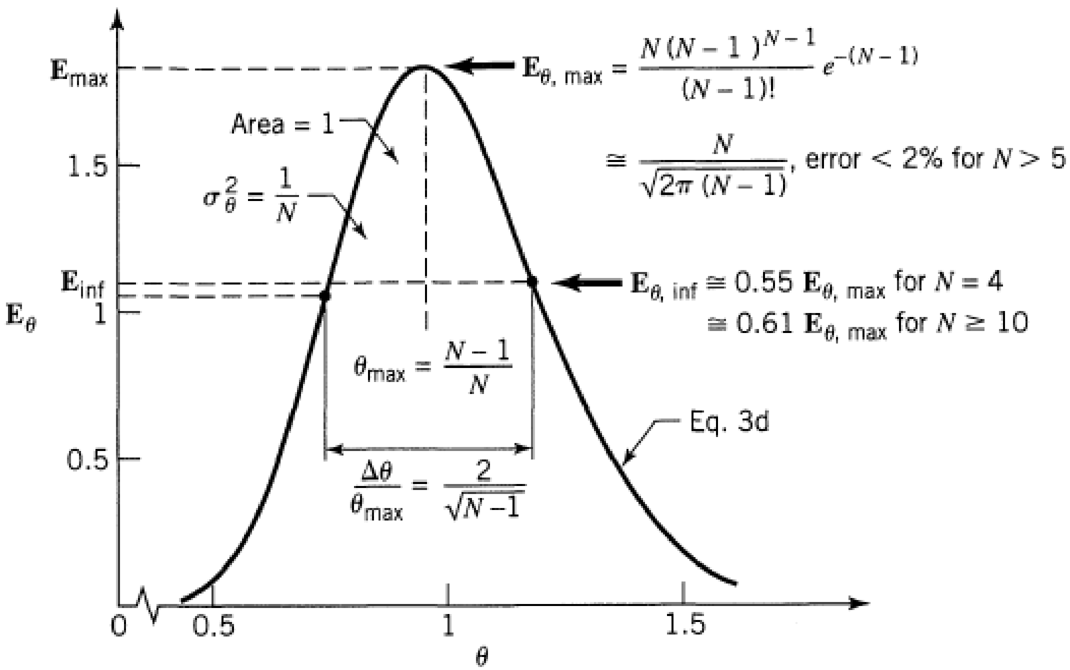
- From the time between peaks

$$\Delta t = \frac{5}{6} = \frac{(1/3)}{(1+1)1} + \frac{V_2}{1(1)} \quad \cdots \underline{V_2 = \frac{2}{3}}$$



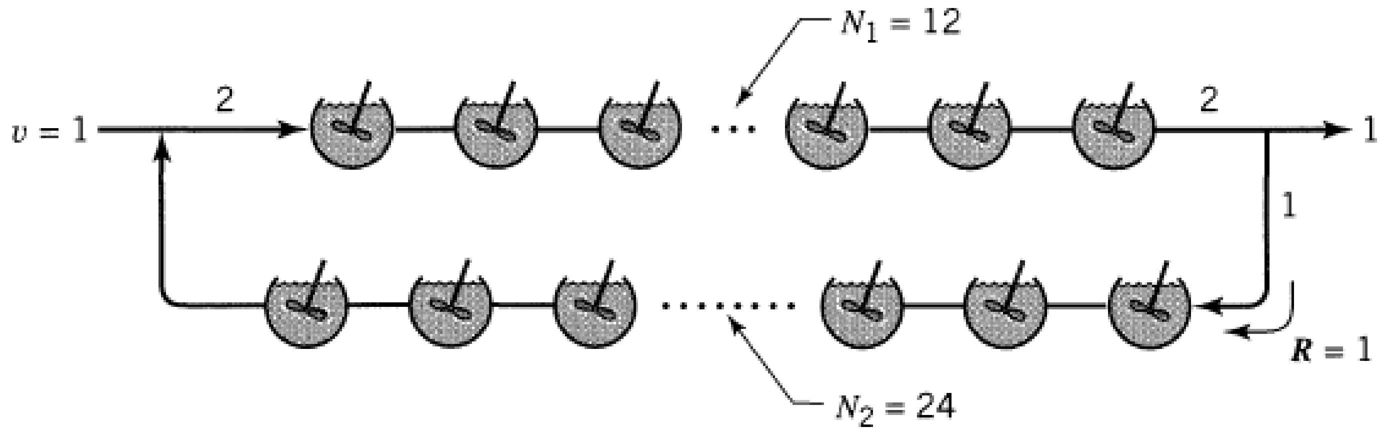
- Since $V_1 + V_2$ add up to 1, there is no dead volume, so at this point our model reduces to Fig. Now relax the plug flow assumption and adopt the tanks-in-series model. From Fig.





$$\frac{\Delta\theta}{\theta_{max}} = \frac{1/10}{1/6} = \frac{2}{\sqrt{N-1}} \quad \cdots N = 12$$

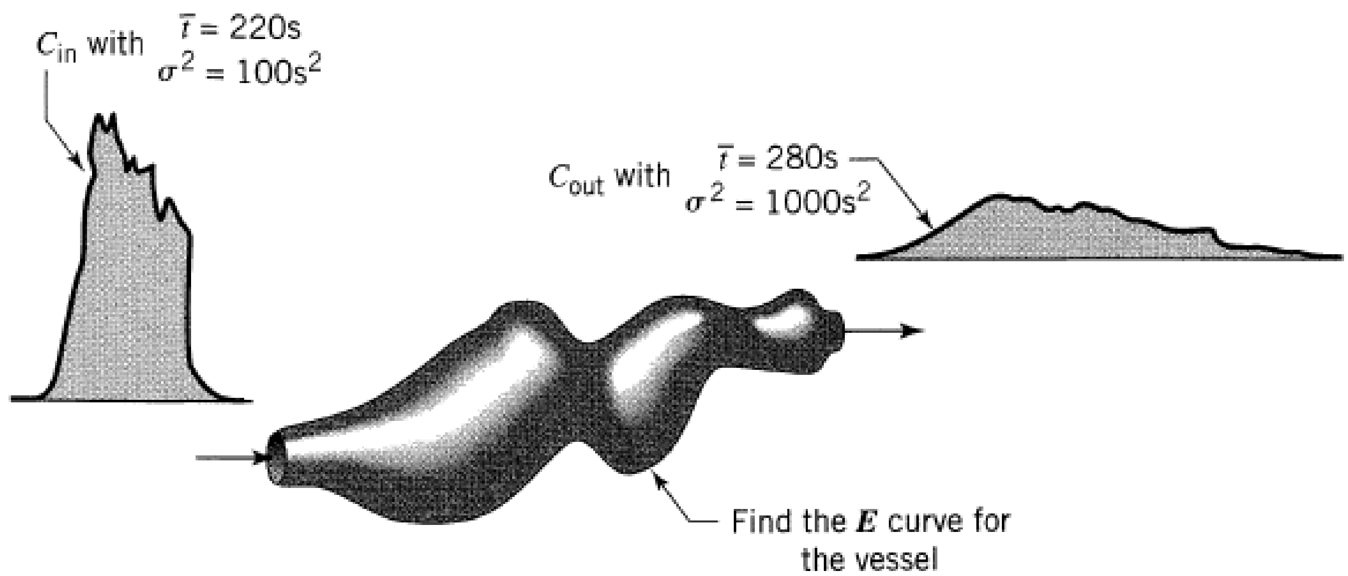
So our model finally is shown in Fig.



FINDING THE VESSEL E CURVE USING A SLOPPY TRACER INPUT

Given C_{in} and C_{out} as well as the location and spread of these tracer curves, as shown in Fig estimate the vessel E curve.

We suspect that the tanks-in series model reasonably represents the flow in the vessel.

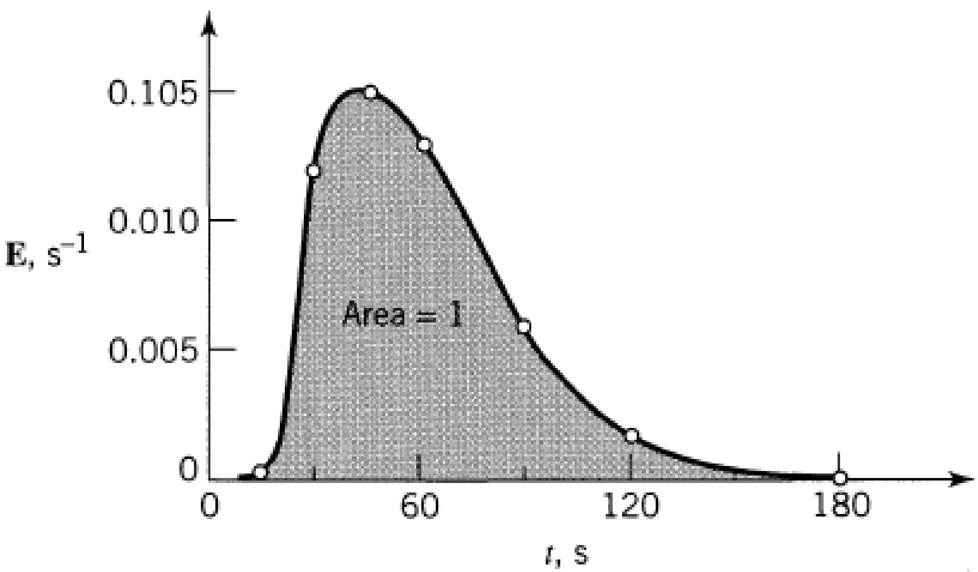


Formula

$$\bar{t}\,\mathbf{E} = \left(\frac{t}{\bar{t}}\right)^{N-1} \frac{N^N}{(N-1)!}\, e^{-tN/\bar{t}} \qquad \cdots \bar{t} = N\bar{t}_i \cdots \sigma^2 = \frac{\bar{t}^2}{N}$$

$$\bar{t}_i\,\mathbf{E} = \left(\frac{t}{\bar{t}_i}\right)^{N-1} \frac{1}{(N-1)!}\, e^{-t/\bar{t}_i} \qquad \cdots \bar{t}_i = \frac{\bar{t}}{N} \cdots \sigma^2 = N\bar{t}_i^2$$

the shape of this E curve.



$$\Delta\bar{t} = 280 - 220 = 60 \text{ s}$$

$$\Delta(\sigma^2) = 1000 - 100 = 900 \text{ s}$$

$$N = \frac{(\Delta\bar{t})^2}{\Delta(\sigma^2)} = \frac{60^2}{900} = 4 \text{ tanks}$$

For N tanks-in-series we have

$$\mathbf{E} = \frac{t^{N-1}}{\bar{t}^{\,N}} \cdot \frac{N^N}{(N-1)!}\, e^{-tN/\bar{t}}$$

and for N = 4

$$\mathbf{E} = \frac{t^3}{60^4} \cdot \frac{4^4}{3 \times 2}\, e^{-4t/60}$$

$$\mathbf{E} = 3.2922 \times 10^{-6}\, t^3\, e^{-0.0667t}$$

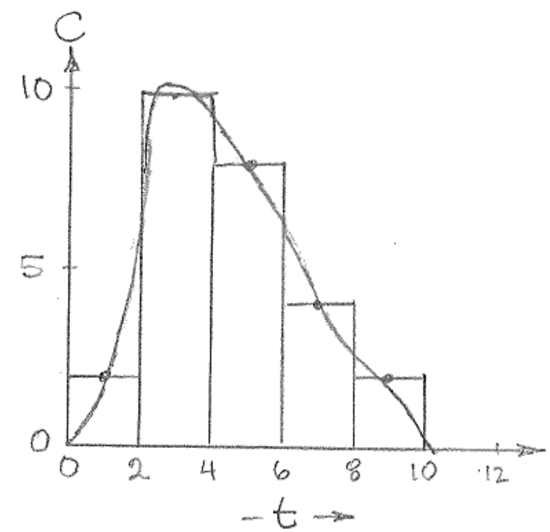
14.1. Fit the tanks-in-series model to the following mixing cup output data to a pulse input.

t	0–2	2–4	4–6	6–8	8–10	10–12
C	2	10	8	4	2	0

14.1 From experiment

t	t_{mean}	C
0-2	1	2
2-4	3	10
4-6	5	8
6-8	7	4
8-10	9	2
10-12	11	0

with linear interpolation



Find the variance & then N

From Eqs 13.1 and 13.3

$$\bar{t} = \frac{\Sigma tC}{\Sigma C} = \frac{1(2)+3(10)+5(8)+7(4)+9(2)}{2+10+8+4+2} = \frac{118}{26} = 4.538$$

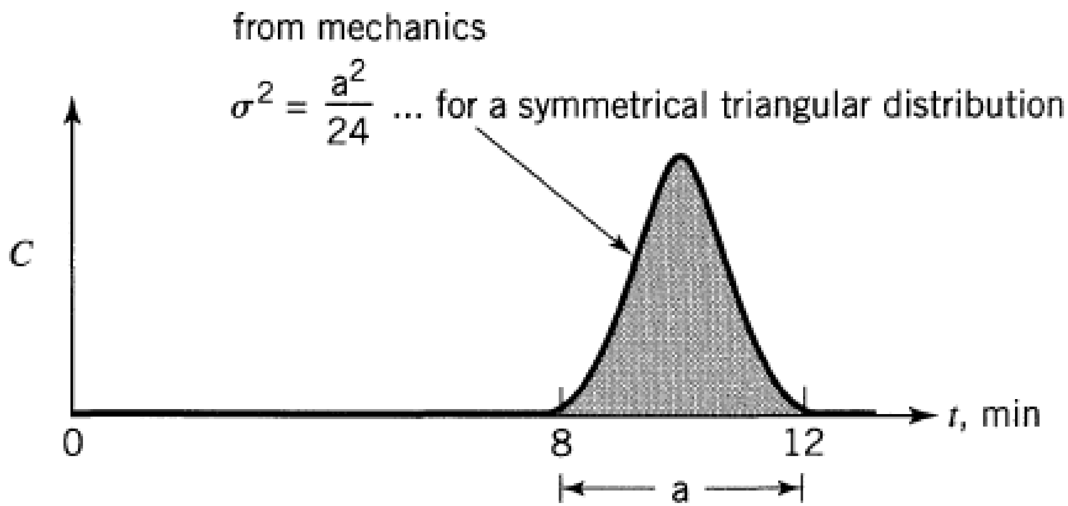
$$\sigma^2 = \frac{\Sigma t^2C}{\Sigma C} - \bar{t}^2 = \frac{1^2(2)+3^2(10)+\cdots}{2+10+\cdots} - (4.538)^2 = 4.4038$$

From Eq 3

$$\frac{1}{N} = \sigma_\theta^2 = \frac{\sigma^2}{\bar{t}^2} = \frac{4.4038}{(4.538)^2} = 0.2138$$

$$\therefore N = \frac{1}{0.2138} = 4.68 \text{ tanks}$$

13.13. Calculations show that a plug flow reactor would give 99.9% conversion of reactant which is in aqueous solution. However, our rector has an RTD somewhat as shown in Fig. P13.13. If $C_{A0} = 1000$, what outlet concentration can we expect in our reactor if reaction is first order? From mechanics $\sigma^2 = a^2/24$ for a symmetrical triangle with base a, rotating about its center of gravity.



14.5. Repeat Problem 13.13, but solve it using the tanks-in-series model instead of the dispersion model.

14.5 For plug flow

$$k\bar{t} = \ln \frac{C_{A0}}{C_A} = \ln \frac{1000}{1} = 6.9078$$

For small deviation from plug flow, by the tanks in series model first calculate σ^2 from the tracer curve. From Fig 13.P13

$$\sigma^2 = \frac{a^2}{24} = \frac{4^2}{24} = \frac{2}{3}$$

$$\bar{t} = 10$$

From Eq 3

$$\frac{1}{N} = \sigma_\theta^2 = \frac{\sigma^2}{\bar{t}^2} = \frac{2/3}{(10)^2} = 0.67 \times 10^{-2} \quad \dots \therefore N = 150 \text{ tanks}$$

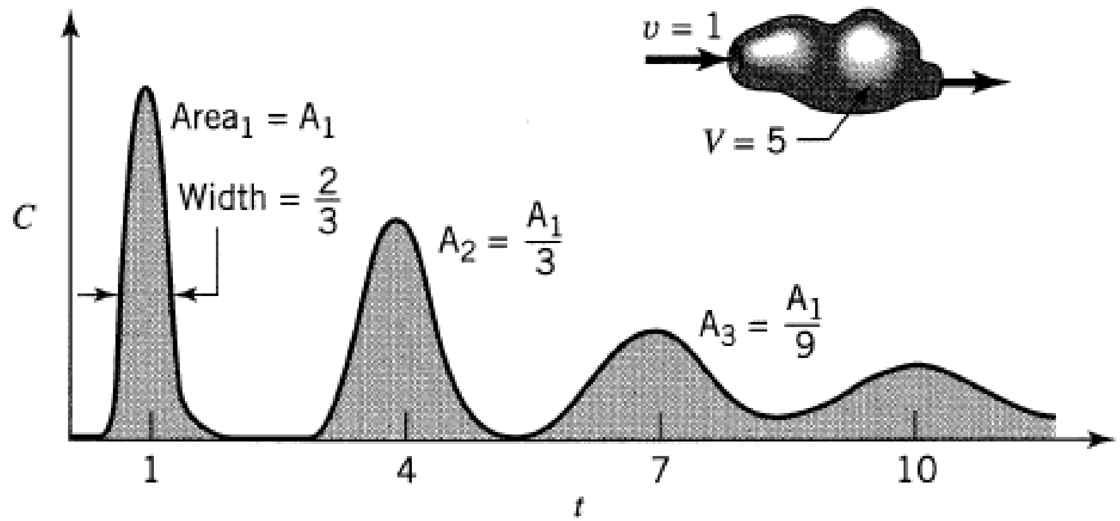
$$\therefore k\bar{t}_i = \frac{6.9078}{150} = 0.0461$$

For tanks in series

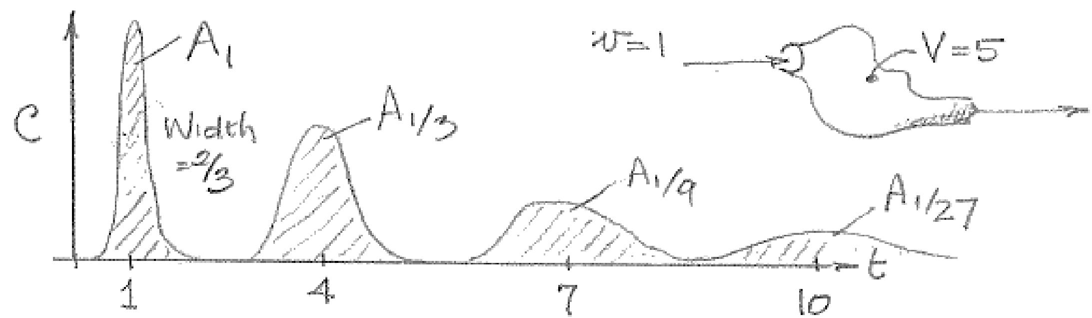
$$\frac{C_A}{C_{A0}} = \frac{1}{(1+k\bar{t}_i)^N} = \frac{1}{(1+0.0461)^{150}} = \frac{1}{863} = 0.00116$$

$$\therefore C_A = 1.16 \longleftarrow$$

14.7. Fit the RTD of Fig. P14.7 with the tanks-in-series model.



14.7



First approximate each pulse by plug flow. Then from Fig 12.1

$$\frac{A_2}{A_1} = \frac{1}{3} = \frac{R}{R+1} \quad \cdots \quad \therefore R = \frac{1}{2}$$

$$\frac{V_{p1}}{(R+1)v} = 1 = \frac{V_{p1}}{(\frac{1}{2}+1)1} \quad \therefore V_{p1} = \frac{3}{2}$$

$$\frac{V_{p1}}{(R+1)v} + \frac{V_{p2}}{Rv} = 3 = \frac{3/2}{(1/2+1)1} + \frac{V_{p2}}{(1/2)1} \qquad \therefore V_{p2} = 1$$

So $V_{active,total} = \frac{3}{2} + 1 = 2.5$

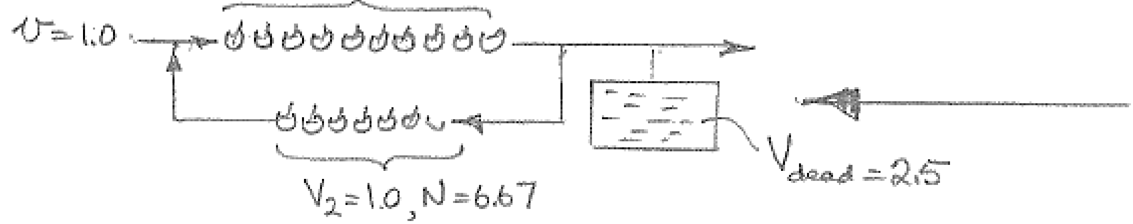
$\therefore V_{dead} = 5 - 2.5 = 2.5$

Now consider that the pulse output has width. Then from Fig 3

$$\frac{\Delta\theta}{\theta_{max}} = \frac{2/3}{1} = \frac{2}{\sqrt{N_1 - 1}} \quad \therefore N_1 = 10 \text{ tanks}$$

and $N_2 = 2/3\, N_1 = 2/3\,(10) = 6.67$ tanks

Thus our model is



Verify

$$\bar{t} = \frac{\Sigma tC}{\Sigma C} = \frac{1(A_1) + 4(A_1/3) + 7(A_1/9) + 10(A_1/27) + 13(A_1/81) + 16(A_1/243) + \cdots}{A_1 + A_1/3 + A_1/9 + A_1/27 + A_1/81 + A_1/243 + \cdots}$$

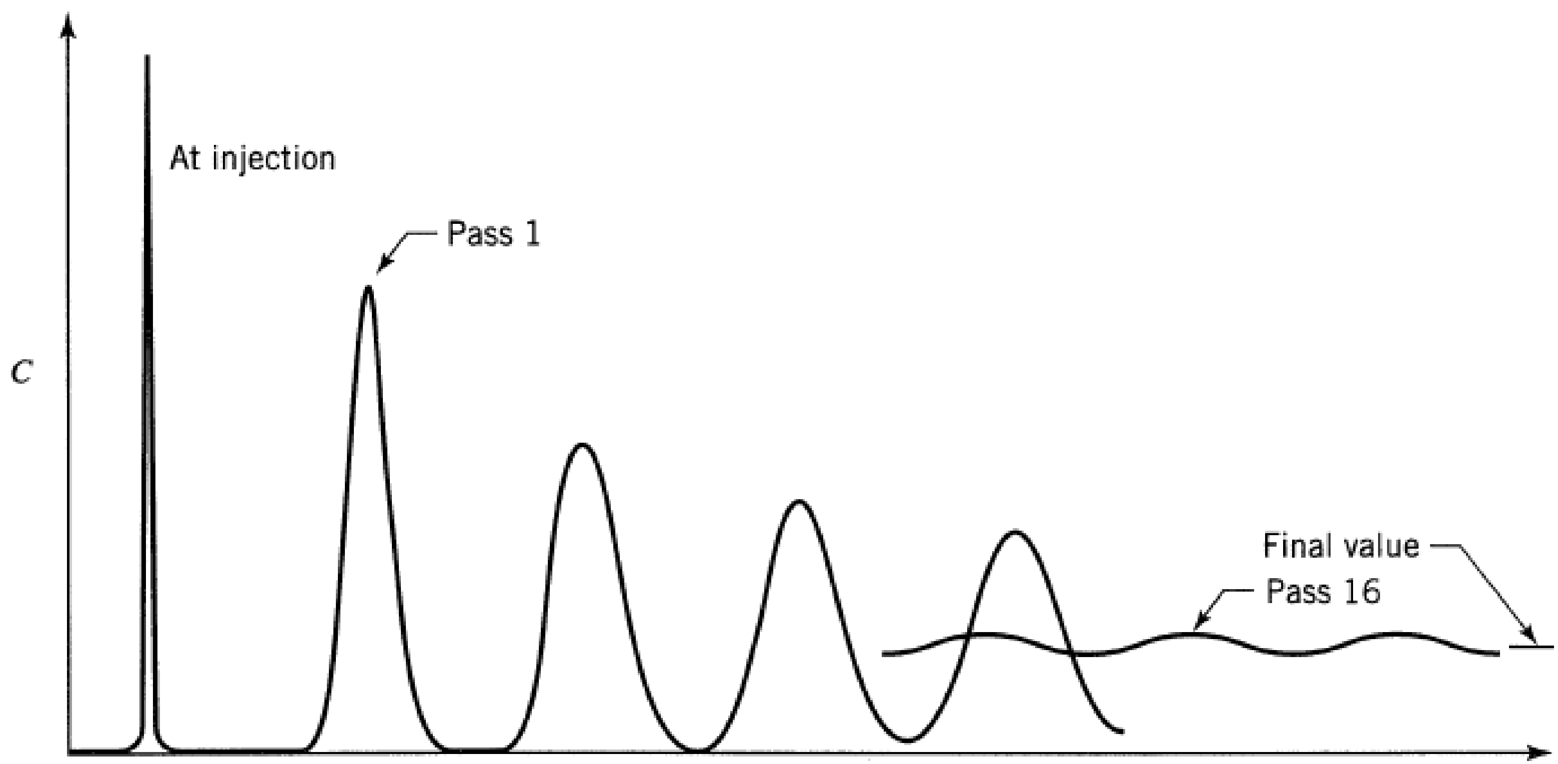
$$= \frac{1 + \frac{4}{3} + \frac{7}{9} + \frac{10}{27} + \frac{13}{81} + \frac{16}{243} + \frac{19}{729} + \cdots}{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \frac{1}{729} + \cdots} = \frac{3.7339}{1.4993} = 2.49 \approx 2.5$$

the limit

This limit equals $\frac{V_1 + V_2}{v} = \frac{1.5 + 1.0}{v} = 2.5$

14.9. Strongly radioactive waste fluids are stored in "safe-tanks" which are simply long, small-diameter (e.g., 20 m by 10 cm) slightly sloping pipes. To avoid sedimentation and development of "hot spots," and also to insure uniformity before sampling the contents, fluid is recirculated in these pipes.

To model the flow in these tanks, a pulse of tracer is introduced and the curve of Fig. P14.9 is recorded. Develop a suitable model for this system and evaluate the parameters.



14.9 For N tanks in series

Fig 25 pg 292 shows that ....

$$N = 1 + 4\left(\frac{\theta_{max}}{\Delta\theta}\right)^2$$

θ_{max} — location of the maximum

$\Delta\theta$ — width of the curve at 60% of its maximum height.

Using a ruler with mm scale and a keen eye we find from Fig P21

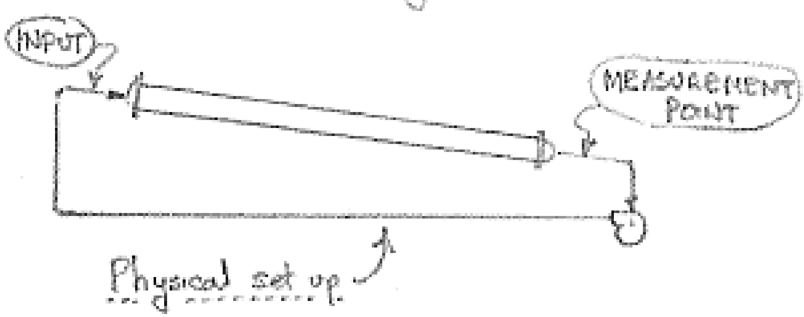
for 1st peak: $N_{1\,pass} = 1 + 4\left(\frac{16.2\,mm}{3.2\,mm}\right)^2 = 104$ tanks ... or $N = 104$ tanks/pass

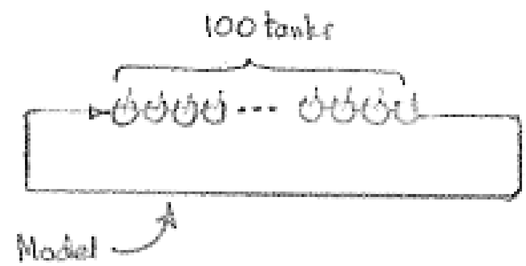
for 2nd peak: $N_{2\,passes} = 1 + 4\left(\frac{32.6}{4.6}\right)^2 = 202$ tanks ... or $N = 101$ tanks/pass

for 3rd peak: $N_{3\,passes} = 1 + 4\left(\frac{48.7}{5.6}\right)^2 = 304$ tanks ... or $N = 101$ tanks/pass

for 4th peak: $N_{4\,passes} = 1 + 4\left(\frac{65}{6.6}\right)^2 = 389$ tanks ... or $N = 97$ tanks/pass

These values average to $N \cong 100$ tanks/pass ←





14.11. A reactor has flow characteristics given by the nonnormalized C curve in Table P14.11, and by the shape of this curve we feel that the dispersion or tanks-in-series models should satisfactorily represent flow in the reactor.

(a) Find the conversion expected in this reactor, assuming that the dispersion model holds.

(b) Find the number of tanks in series which will represent the reactor and the conversion expected, assuming that the tanks-in-series model holds.

(c) Find the conversion by direct use of the tracer curve.

(d) Comment on the difference in these results, and state which one you think is the most reliable.

Data. The elementary liquid-phase reaction taking place is A + B $\rightarrow$ products, with a large enough excess of B so that the reaction is essentially first order. In addition, if plug flow existed, conversion would be 99% in the reactor.

Table P14.11.

Time	Tracer Concentration	Time	Tracer Concentration
0	0	10	67
1	9	15	47
2	57	20	32
3	81	30	15
4	90	41	7
5	90	52	3
6	86	67	1
8	77	70	0

14.11 To find the non ideal characteristics of the experimental reactor determine the proper D/uL to use for the dispersion model, or the proper N value to use for the tanks-in-series model.

This is done in one of two ways — by matching the experimental tracer curve with the family of curves shown in Fig 13.9 (for the dispersion model) or with Fig 2 (for the tanks-in series model), or by calculating σ_θ^2, and from that D/uL or N.

Let us use the latter procedure. So first calculate $\bar{t}$ and σ^2 from the table of data with Eqs. 13.1 and 13.3. This gives

$\Sigma C = 213$ $\qquad \bar{t} = 2149/213 = 10.09$ min

$\Sigma tC = 2149$ $\qquad \sigma^2 = 37685/213 - (10.09)^2 = 75.1168$

$\Sigma t^2 C = 37685$

$$\therefore \sigma_\theta^2 = \frac{\sigma^2}{\bar{t}^2} = \frac{75.1168}{(10.09)^2} = 0.7378$$

Next determine the behavior in an ideal plug flow reactor

$k = 0.456\ \text{min}^{-1}$ $\quad k\bar{t} = (0.456)(10.09) = 4.6$

So for plug flow $\quad X_A = 1 - e^{-kt} = 1 - e^{-4.6} = 0.99$

Now we are ready to proceed with our problem

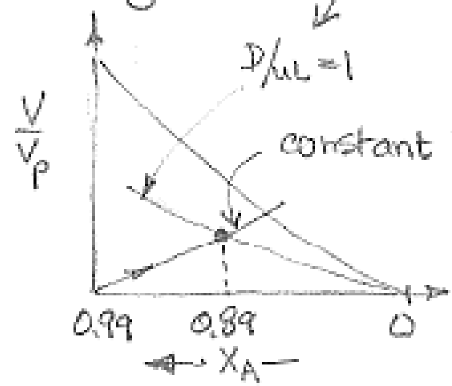
(a) Use the dispersion model

Here Eq. 13.15 relates σ_θ^2 with D/uL. So

$$\sigma_\theta^2 = 0.7378 = 2\left(\frac{D}{uL}\right) - 2\left(\frac{D}{uL}\right)^2\left[1 - e^{-uL/D}\right]$$

Solve by trial and error. This gives $D/uL = 1$

Then from Fig. 13.19



From this figure

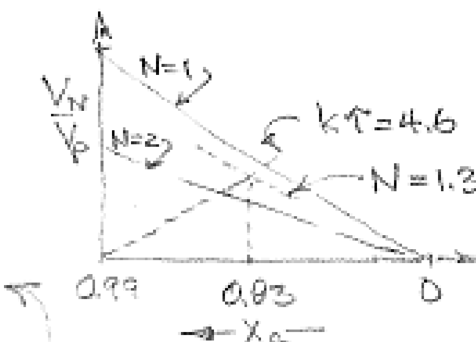
$X_{disp.} = 0.89$ ← a)

14.11 (continued) (b) Use the tanks-in-series model

From Eq. 3 we find

$$N = \frac{1}{\sigma_\theta^2} = \frac{1}{0.7388} = 1.3554 \text{ tanks}$$

So from Fig. 6.5 we find $X_{tanks} = 83\%$ ← b)



(c) Use the tracer data directly.

From Eq 11.13 $\frac{C}{C_0} = \sum \left(\frac{C}{C_0}\right)_{batch} E\Delta t$

To find the E curve make the area under the C curve unity, or as shown in Example 11.1

$$E = \frac{C}{area} = \frac{C}{\Sigma C_i \Delta t} \quad \cdots \text{ or } \quad E\Delta t = \frac{C}{\Sigma C}$$

t, min	C	$e^{-0.456 t_i} C_i / 213$
0	0	0
1	17	0.0506
3	38	0.0454
5	33	0.0158
7	26	0.0050
9	20	0.0015
11	16	0.0005
13	13	0.0002
15	10	0.0001
17	8	16×10^{-6}
19	6	5×10^{-6}
21	5	2×10^{-6}
23	4	1×10^{-6} } = 0
25-27	3	-
29-31	2	-
33-45	1	-
	$\Sigma C = 213$	$\Sigma = 0.1191$

Now follow the method of Example 11.4

$$\frac{C}{C_0} = \Sigma \left(\frac{C}{C_0}\right)_{batch} E\Delta t$$

$$= \Sigma e^{-kt} \cdot \frac{C}{\Sigma C}$$

$$= \sum_i e^{-0.456 t_i} \cdot \frac{C_i}{213}$$

now tabulate

$X_{\text{from curve}} = 0.88$ ← c)

(d) Which answer is most reliable

Naturally the direct use of the tracer curve gives the most reliable answer. In this problem the given RTD came from the dispersion model with $D/uL = 1$. Thus we'd expect that the answers to parts a) and c) should agree. They do.