COMPLEX VARIABLES

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \text{when } 4AC > B^2$$

$$\frac{-B \pm i\sqrt{4AC - B^2}}{2A} \quad (i = \sqrt{-1})$$

this is defined due to the fact that every quadratic equation must have two roots.

A complex variable $z = x + iy$ is an ordered pair of real variables x and y which satisfy certain law of operation.

In general $x + iy \neq y + ix$

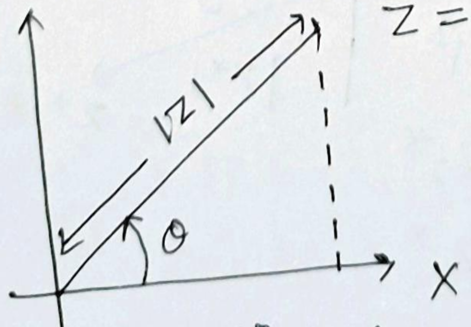
Real parts of $z \Rightarrow x$
$Re(z)$

imaginary parts of z $Im(z) \Rightarrow y$.

(i) If $z = 0$ then $x = y = 0$

(ii) If $z_1 = z_2$ $x_1 = x_2$
$y_1 = y_2$.

Argand Diagram $\rightarrow$



$z = x + iy$

$(x, y) \Leftrightarrow (r, \theta)$

$x = r\cos\theta, \quad y = r\sin\theta$

$z = r\cos\theta + iy\sin\theta\, r = r(\cos\theta + i\sin\theta)$
$= re^{i\theta}$

$r = \sqrt{x^2 + y^2} = |z| = \text{Mod} z$, $\theta = \arg z$

① $z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$

$= (x_1 + x_2) + i(y_1 + y_2)$

② $z_1 - z_2 = (x_1 + iy_1) + (x_2 + iy_2)$

$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$

③ $$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \times \frac{x_2 - iy_2}{x_2 - iy_2}$$

$$\frac{z_1}{z_2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\left\{\frac{x_2 y_1 - x_1 x_2}{x_2^2 + y_2^2}\right\}$$

$z_2 \neq 0$

④ Associative law of addition.

$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

⑤ Commutative law of multiplication.

$z_1 z_2 = z_2 z_1$

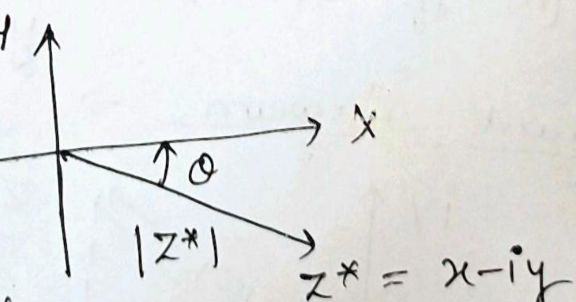
⑥ Associative law of multiplication.

$z_1(z_2 z_3) = (z_1 z_2) z_3$

⑦ Distributive law -

$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$

complex conjugate - $z^* = x - iy$.



$zz^* = |z|^2$

$(x + iy)(x - iy)$

$= x^2 + y^2 = |z|^2$

$(z_1 + z_2)^* = z_1^* + z_2^*$

$(z_1 z_2)^* = z_1^* z_2^*$.

$$\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$$

Euler's formula -

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$$x = i\theta$$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!}$$

$$= 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} - \frac{\theta^4}{4!} - \frac{i\theta^5}{5!}$$

$$= \underbrace{i\theta - \frac{i\theta^3}{3!} + \frac{i\theta^5}{5!}}_{i \sin\theta} + 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}$$

$$i \sin\theta \quad + \cos\theta$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

De Moivere's theorem -

$$z^n = r^n(\cos n\theta + i \sin n\theta)$$