Continuous case: Let X be continuous random variable.

Ex: If we ~~take~~ measure height of people in Kanpur then instead of having discrete values like 5 ft or 6 ft, ~~they~~ values like 5.1 ft, 5.125 ft, ~~4~~.95 ft etc will come. Hence in such cases the random variable is taken as continuous random variable.

Probability density function (pdf): A fn $f(x)$ can be called as pdf of continuous random variable X, if it satisfies

(1) $f(x) \geq 0$ $\quad -\infty \leq x \leq \infty$

(2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

Note) $P(a < X < b) = \int_a^b f(x)\,dx$.

cumulative Distribution fn: of continuous random variable X is defined by $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx \quad -\infty < x < \infty$.

Note: $F'(x) = f(x)$.

Ex Find constant c so that $f(x) = \begin{cases} 4x^c, & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ is pdf of X. Also find cumulative distribution fn.

Soln (1) $f(x) \geq 0$ (the way $f(x)$ is defined

(2) $1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 4x^c\,dx = 4\left.\frac{x^{c+1}}{c+1}\right|_0^1 = 4\left[\frac{1}{c+1}\right] \Rightarrow \boxed{c=3}$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx .$$

For $x < 0$ $f(x) = 0$ $\therefore F(x) = 0$.

For $x \le 1$ $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} 4x^3\,dx = x^4$

For $x > 1$ $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{1} f(x)\,dx + \int_{1}^{x} \cancel{f(x)}^{0}\,dx$

$= 1$.

$$\therefore F(x) = \begin{cases} 0 & x < 0 \\ x^4 & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

Some important points: For continuous random variable X:

- $P(X = a) = 0$.
- $P(a \le X \le b) = P(a < X \le b) = P(a \le X < b)$
  $= P(a < X < b) = \int_{a}^{b} f(x)\,dx$.
- $F(x)$ is continuous function of x on right
- $F(-\infty) = 0$ & $F(\infty) = 1$.
- $F'(x) = f(x) \ge 0$ $\therefore F(x)$ is non-decreasing.