Discrete Case:

Random Variables (X): $X: S \to \mathbb{R}$, where S is sample space corresponding to random experiment & $\mathbb{R}$ is real number.

Ex I Consider the random experiment of tossing two coins (unbaised). Then sample space $= \{HH, HT, TH, TT\}$

(1) $X \to$ no. of heads

Then $X(HH) = 2$ $\quad X(HT) = 1$ $\quad X(TH) = 1$ $\quad X(TT) = 0$

$\therefore X \to 0, 1, 2.$

(2) $Y \to$ no. of heads $-$ no. of tails

Then $Y(HH) = 2$ $\quad Y(HT) = 0$ $\quad Y(TH) = 0$ $\quad Y(TT) = -2$

$\therefore Y \to -2, 0, 2.$

Probability Mass function / Probability function:

Define $f(x) = P(X = x)$. Then $f(x)$ is pmf / prob fn, provided it satisfies

(1) $f(x) \geq 0 \quad \forall x.$

(2) $\sum_x f(x) = 1.$

Ex II In example I above for random variable X

$f(0) = P(X = 0) = 1/4$ $\qquad f(1) = P(X = 1) = 1/2$

$f(2) = P(X = 2) = 1/4$

$\therefore$

X	0	1	2
f(x)	1/4	1/2	1/4

clearly $f(x) \geq 0$

& $\sum_x f(x) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1.$

Ex A pair of die is thrown. Let random variable X be sum on dice ~~[illegible]~~. Find pmf.

Soln

X	2	4	6	8	10	12	3	5	7	9	11
$f(x)$	$1/36$	$3/36$	$5/36$	$5/36$	$3/36$	$1/36$	$2/36$	$4/36$	$6/36$	$4/36$	$2/36$

Distribution function: $F(x)$ is defined by

$$F(x) = P(X \le x) = \sum_{t \le x} f(t) \quad -\infty < x < \infty$$

Properties: (1) $0 \le F(x) \le 1$

(2) $F(x)$ is non-decreasing function of x i.e, if $b \ge a$ then $F(b) \ge F(a)$

(3) $F(b) - F(a) = P(X \le b) - P(X \le a) = P(a < X \le b)$

(4) $P(a < x < b) = F(b) - F(a) - P(X=b)$

(5) $P(a \le x \le b) = F(b) - F(a) + P(X=a)$

(6) $P(a \le x < b) = F(b) - F(a) + P(X=a) - P(X=b)$

(7) $F(x)$ is right continuous at the points of discontinuities, $x_1, x_2, \ldots x_n$ i.e, $F(x_i+0) = F(x_i)$

Theorem: If random variable X takes values $x_1, x_2, \ldots x_n$ then $f(x_j) = P(X = x_j) = F(x_j) - F(x_{j-1})$.

where $x_1 < x_2 < \ldots < x_n$

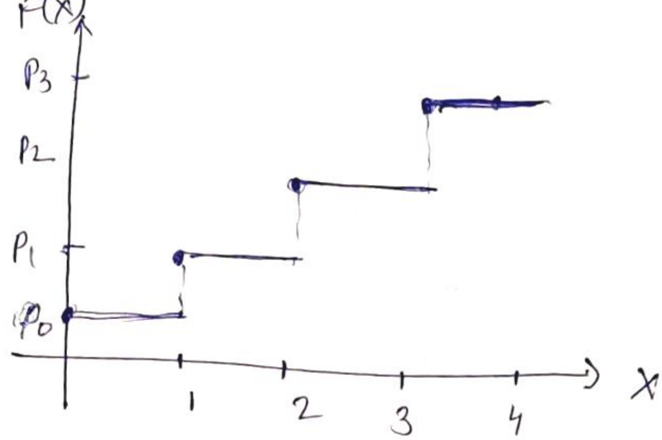
Proof! $F(x_j) = P(X \le x_j)$ (definition

$$= \sum_{i=1}^{j} P(X = x_i) = \sum_{i=1}^{j} f(x_i)$$

Therefore $F(x_{j-1}) = P(X \le x_{j-1}) = \sum_{i=1}^{j-1} f(x_i)$

& $F(x_j) - F(x_{j-1}) = f(x_j)$ //.

Graph of $F(x)$:



Ex III For Ex I find
(1) Distribution fn
(2) $P(X \le 2)$, $P(X > 1)$ using distribution fn.
(3) $P(X = 1)$

Soln (1)

X	0	1	2
F(x)	1/4	3/4	1

$F(0) = f(0) = P(X \le 0) = 1/4$
$F(1) = P(X \le 1) = 3/4$
$F(2) = P(X \le 2) = 1$.

(2) $P(X \le 2) = F(2) = 1$

$P(X > 1) = 1 - P(X \le 1) = 1 - F(1) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(X = 1) = F(X = 1) - F(X = 0)$ (above thm)

$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.