22/8/22 Lecture-11

Questions on S_3 :-

① How many transpositions are there in S_3? Ans → 3

② How many odd permutations in S_3? = 3

③ How many elements σ in S_3 such that $\sigma^4 = I$? ⇒ 1+3 = 4

④ How many elements in S_3 such that $\sigma^3 = I$ but σ is not identity? ⇒ 2

⑤ How many elements σ in S_3 such that $\sigma^5 = I$? ⇒ 1

⑥ How many elements σ in S_3 such that σ is non-identity and $\sigma^5 = I$? ⇒ 0

Q. How many types of elements in S_3?
No. of partitions of 3 = P(3) = 3

Q. Formula to find the no of elements of a particular type in S_3.

$3 = 3$
$3 = 2+1$
$3 = 1+1+1$

$S_3 = \{I, (1\,2), (1\,3), (2\,3), (1\,2\,3), (1\,3\,2)\}$

Identity type ⇒ no. of elements $= \frac{3!}{1^3 \cdot 3!} = 1$

Transposition type → $\frac{3!}{2^1 \cdot 1! \cdot 1! \cdot 1} = \frac{3!}{2} = 3$

Length 3 type → $\frac{3!}{3^1 \cdot 1!} = 2$

S_3

S.No.	Partition	Type	No. of elements	Even/Odd	Order
1	3	(1 2 3)	$\frac{3!}{3!\,1!} = 2$	3-1=2 even	3
2	2+1	(1 2)	$\frac{3!}{2!1!\cdot1!\,1!} = 3$	3-2=1 odd	lcm(2,1) = 2
3	1+1+1	I	$\frac{3!}{1^3 3!} = 1$	3-3=0 even	lcm(1,1,1) = 1

S_4

S.No	Partition	Type	No. of elements	Order	Even/Odd
1	4	(1 2 3 4)	$\frac{4!}{4!\,1!} = 6$	4	4-1=3 Odd
2	3+1	(1 2 3)(4)	$\frac{4!}{3!1!\cdot1!\,1!} = 8$	lcm(3,1) = 3	4-2=2 Even
3	2+2	(1 2)(3 4)	$\frac{4!}{2^2 2!} = 3$	lcm(2,2) = 2	4-2=2 Even
4	2+1+1	(1 2)(3)(4)	$\frac{4!}{2!1!\cdot1^2 2!} = 6$	lcm(2,1,1) = 2	4-3=1 Odd
5	1+1+1+1	I	1	lcm(1,1,1,1) = 1	4-4=0 Even