Finite Fourier Transform.

① finite fourier Sine transform

The finite fourier Sine transform of $F(x)$, $0 < x < l$, is denoted by $f_s(s)$ or $F_s\{F(x)\}$ and is defined by

$$f_s(s) = F_s\{F(x)\} = \int_0^l F(x) \sin\left(\frac{s\pi x}{l}\right) dx$$

where s is positive integer

and its inversion formula is

$$F_s^{-1}\{f_s(s)\} = F(x) = \frac{2}{l} \sum_{s=1}^{\infty} f_s(s) \sin\left(\frac{s\pi x}{l}\right)$$

② Finite Fourier Cosine transform

The finite fourier cosine transform of $F(x)$, $0 < x < l$ is defined by

$$f_c(s) = F_c\{F(x)\} = \int_0^l F(x) \cos\left(\frac{x\pi s}{l}\right) dx$$

where s is zero and +ve integers.

and the inversion formula is

$$F_c^{-1}\{f_c(s)\} = F(x) = \frac{1}{l} f_c(0) + \frac{2}{l} \sum_{s=1}^{\infty} f_c(s) \cos\left(\frac{s\pi x}{l}\right)$$

Ex. 1 find the finite Fourier transform of

$$F(x) = x^2, \quad 0 < x < 4$$

Soln. Here $l = 4$

The finite Fourier Sine transform.

$$F_s\{x^2\} = \int_0^4 x^2 \sin\left(\frac{\pi s x}{4}\right) dx$$

$$= \left| -x^2 \cos\left(\frac{s\pi x}{4}\right)\cdot\left(\frac{4}{\pi s}\right) + 2x \sin\left(\frac{s\pi x}{4}\right)\left(\frac{4}{\pi s}\right)^2 + 2\cos\left(\frac{s\pi x}{4}\right)\left(\frac{4}{\pi s}\right)^3 \right|_0^4$$

$$= \frac{-64}{\pi s}\cos s\pi + 0 + \frac{128}{\pi^3 s^3}\cos s\pi - 0 - 0 - \frac{128}{\pi^3 s^3}$$

$$\therefore F_s\{x^2\} = \frac{-64}{\pi s}(-1)^s + \frac{128}{\pi^3 s^3}\{(-1)^s - 1\}$$

and the finite Fourier cosine transform

$$F_c\{x^2\} = \int_0^4 x^2 \cos\left(\frac{s\pi x}{4}\right)dx$$

$$= \left| x^2 \sin\left(\frac{s\pi x}{4}\right)\left(\frac{4}{s\pi}\right) + 2x\cos\left(\frac{s\pi x}{4}\right)\left(\frac{4}{s\pi}\right)^2 - 2\sin\left(\frac{s\pi x}{4}\right)\left(\frac{4}{s\pi}\right)^3 \right|_0^4$$

$$= 0 + \frac{128}{s^3\pi^3}\cos s\pi - 0$$

$$\therefore F_c\{x^2\} = \frac{128(-1)^s}{s^3\pi^3}$$

Ex.2 Find $F(x)$ if $f_c(s) = \dfrac{\cos(2s\pi/3)}{(2s+1)^2}$

Soln: By inversion formula

$$F(x) = F_c^{-1}\{f_c(s)\} = \frac{1}{\pi}f_c(0) + \frac{2}{\pi}\sum_{s=1}^{\infty} f_c(s)\cos sx$$

$$= \frac{1}{\pi} + \frac{2}{\pi}\sum_{s=1}^{\infty}\frac{\cos(2s\pi/3)}{(2s+1)^2}\cos sx$$