Theorem:- A subset G of R is open iff it is a union of open intervals.

Proof:- We assume that G is open.
let $\{G_i\}$ be the collection of open intervals contained in G.

We have to show that $G = \bigcup_{i \in I} G_i$

Obviously $\bigcup_{i \in I} G_i \subseteq G$ —— ①

further, if $x \in G$, since G is open then there exists some open interval, $x \in G_{i_0}$ in $\{G_i\}$.

Clearly, $x \in G_{i_0} \subseteq \bigcup_{i \in I} G_i$

$\Rightarrow G \subseteq \bigcup_{i \in I} G_i$ —— ②

From ① & ②
we get $G = \bigcup_{i \in I} G_i$

Conversely, let G be a union of open interval.
Since each open interval is open set, G is a union of open sets.
Hence G is open

Ex- (i) Give an example of each of the following
(i) An open set which is not an open interval
$G = (0,1) \cup (2,3)$
(ii) An interval which is not an open set.
Any close interval.
Ex - $[0,1]$

(iii) A set which is neither an interval nor an open set.

$[0,1] \cup [2,3]$

any finite set

(iv) It is not necessary for a super set to be open set.

(v) ~~Op~~ Any interval is not open set.

(vi) Set of Real numbers is open.

Ques:- Show that the intersection of an arbitrary collection of open sets may or may not be an open set