

20/8/22

Lecture-10

S_4 : is the collection of all functions from X to X where $X = \{1, 2, 3, 4\}$

$S_4 = \{f: X \rightarrow X, X = \{1, 2, 3, 4\}, f \text{ is one-one onto}\}$
 $|S_4| = 4! = 24$

Order of elements of $S_4 = 1/2/3/4/6/8/12/24$

$S_4 = \{I, (1\ 2), (1\ 3), (1\ 4), (2\ 3), (2\ 4), (3\ 4),$
 $(1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3),$
 $(1\ 2\ 3), (1\ 2\ 4), (1\ 3\ 4), (2\ 3\ 4),$
 $(1\ 3\ 2), (1\ 4\ 2), (1\ 4\ 3), (2\ 4\ 3),$
 $(1\ 2\ 3\ 4), (1\ 3\ 2\ 4), (1\ 4\ 2\ 3), (1\ 2\ 4\ 3),$
 $(1\ 3\ 4\ 2), (1\ 4\ 3\ 2)\}$

Partition of an Integer:-

Let 'n' be a positive integer then a finite sequence of positive integers n_1, n_2, \dots, n_k is called a partition of n if —

$$n = n_k + n_{k-1} + \dots + n_2 + n_1$$

where, $n_k \geq n_{k-1} \geq \dots \geq n_2 \geq n_1 > 0$

$P(n) \rightarrow$ no. of partitions

① $n=1 \quad 1=1$

$P(1) = 1$

② $n=2 \quad 2=2$

$2=1+1$

$P(2) = 2$

$$\textcircled{3} \quad n=3 \quad 3=3$$

$$3=2+1$$

$$3=1+1+1$$

$$P(3)=3$$

$$\textcircled{4} \quad n=4 \quad 4=4$$

$$4=3+1$$

$$4=2+1+1$$

$$4=2+2$$

$$4=1+1+1+1$$

$$P(4)=5$$

$$\textcircled{5} \quad n=5 \quad 5=5$$

$$5=4+1$$

$$5=3+1+1$$

$$5=3+2$$

$$5=2+1+1+1$$

$$5=1+1+1+1+1$$

$$5=2+2+1$$

$$P(5)=7$$

$$\textcircled{6} \quad n=6 \quad 6=6$$

$$6=5+1$$

$$6=4+1+1$$

$$6=4+2$$

$$6=3+1+1+1$$

$$6=3+3$$

$$6=3+2+1$$

$$6=2+1+1+1+1$$

$$6=2+2+2$$

$$6=1+1+1+1+1+1$$

$$6=2+2+1+1$$

$$P(6)=11$$

⑦ $n=7$

$P(7)=15$

$7=7$

$7=6+1$

$7=5+1+1$

$7=5+2$

$7=4+3$

$7=4+2+1$

$7=4+1+1+1$

$7=3+3+1$

$7=3+2+2$

$7=3+2+1+1$

$7=3+1+1+1+1$

$7=2+2+2+1$

$7=2+2+1+1+1$

$7=2+1+1+1+1+1$

$7=1+1+1+1+1+1+1$

$P(8)=22$

$P(9)=30$

$P(10)=42$

★ length 2 cycle is known as transposition.

Even Permutation: A permutation is said to be even permutation if it can be written as product of even number of transpositions.

Odd Permutation: A permutation is said to be odd permutation if it can be written as product of odd number of transpositions.

Ex → ① $(1\ 2\ 3\ 4) = (1\ 4)(1\ 3)(1\ 2)$
Odd Permutation

② $(1\ 2\ 3) = (1\ 3)(1\ 2)$
Even Permutation

③ For disjoint cycles

Ex - $(1\ 2\ 3)(7\ 8) = \text{Odd Permutation}$

$$\begin{array}{ccccc} + & - & = & - \\ \text{Even} & \text{Odd} & & \text{Odd} \end{array}$$

Ex

$(1\ 2\ 3)(7\ 8) = (1\ 3)(1\ 2)(7\ 8)$
Odd Permutation