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Q Show that the arbitrary collection of open sets may or may not be open.

(i) For each $n \in N$, let $G_n = (0, n)$. Then each G_n is open set.

$$\bigcap_{n=1}^{\infty} G_n = (0,1) \cap (0,2) \cap (0,3) \cap \ldots \cap (0,\infty)$$
$$= (0,1), \text{ which is open.}$$

(ii) For each $n \in N$, let $G_n = \left(\frac{-1}{n}, \frac{1}{n}\right)$. Then each G_n is open set. Further,

$$\bigcap_{n=1}^{\infty} G_n = (-1,1) \cap \left(\frac{-1}{2}, \frac{1}{2}\right) \cap \left(\frac{-1}{3}, \frac{1}{3}\right) \cap \ldots\ldots$$
$$= \{0\}$$

which is finite set. Hence, it is not open.

Limit Points:- Let E be a subset of R.
A point 'p' $\in R$ is called a limit point of E if every deleted nbd of p contains atleast one point of p.

or

$$(N_p - \{p\}) \cap E \neq \phi$$

Eg → ① If $A = (0,1)$ be a subset of $B = [0,1]$ then every point of $[0,1]$ is the limit point of $(0,1)$.

② Every point of $[0,1]$ is a limit point of it and no other points can be its limit point.

③ Any finite set has no limit point.

④ The set N has no limit point even though it is a infinite set.

⑤ $\left\{\frac{1}{n} : n \in N\right\}$ has '0' as its limit point.