29/8/22

Theorem: let A and B be any two subsets of $\mathbb{R}$, then —

(i) $D(\phi) = \phi$

(ii) $A \subseteq B \Rightarrow D(A) \subseteq D(B)$

(iii) $D(A \cup B) = D(A) \cup D(B)$

(iv) $D(A \cap B) \subseteq D(A) \cap D(B)$

(v) $x \in D(A) \Rightarrow x \in D(A - \{x\})$

Proof → (i) If 'p' be any point of $\mathbb{R}$, then $\mathbb{R}$ is a nbd of 'p' such that —

$$\mathbb{R} \cap \phi = \phi$$

$\Rightarrow$ p is not limit point of ϕ.

Thus no point of $\mathbb{R}$ can be limit point of ϕ.

Hence, $D(\phi) = \phi$

—— Proved

(ii) let $A \subseteq B$, then let $x \in D(A)$

$\Rightarrow$ every deleted nbd of x contains a point of A.

$\Rightarrow$ every deleted nbd of x contains a point of B.

$\Rightarrow$ x is also a limit point of B.

$\Rightarrow x \in D(B)$

Hence, $D(A) \subseteq D(B)$

—— Proved

(iii) Since, $A \subseteq A \cup B$ and $B \subseteq A \cup B$

$\Rightarrow D(A) \subseteq D(A \cup B)$ and $D(B) \subseteq D(A \cup B)$

$\Rightarrow D(A) \cup D(B) \subseteq D(A \cup B)$ —— ①

Further, $x \in D(A \cup B)$
$\Rightarrow$ every deleted nbd of x contains atleast one point of $A \cup B$.
$\Rightarrow x \in D(A)$ or $x \in D(B)$
$\Rightarrow x \in D(A) \cup D(B)$
Hence, $D(A \cup B) \subseteq D(A) \cup D(B)$ ——— ②

from eq-① & ②, we get $\rightarrow$
$$D(A \cup B) = D(A) \cup D(B)$$
——— Proved

(iv) Since, $A \cap B \subseteq A$ and $A \cap B \subseteq B$
$\Rightarrow D(A \cap B) \subseteq D(A)$ and $D(A \cap B) \subseteq D(B)$
$\Rightarrow D(A \cap B) \subseteq D(A) \cap D(B)$
——— Proved

(v) We have $x \in D(A)$
$\Rightarrow x$ is a limit point of A
$\Rightarrow$ every deleted nbd of x contains a point of A
$\Rightarrow$ every deleted nbd of x contains a point of $(A - \{x\})$
$\Rightarrow x \in D(A - \{x\})$
——— Proved