

Q Determine the limit points of the set  $N$ .  
Ans  $\rightarrow D(N) = \phi$

Q Determine the limit points of the following sets —

- (i)  $[a, b]$
- (ii)  $R - a$
- (iii) Finite set

- (i)  $D\{[a, b]\} = [a, b]$
- (ii)  $D\{R - a\} = R$
- (iii)  $D\{\text{Finite set}\} = \phi$

Q Give example of set  $E$  such that —

(i)  $E \cap D(E) = \phi$

let  $E = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \dots \right\}$  Ans

$$D(E) = \{0\}$$

$$E \cap D(E) = \phi$$

(ii)  $E \subset D(E)$

let  $E = (a, b)$  Ans

$$D\{(a, b)\} = [a, b]$$

$$\Rightarrow E \subset D(E)$$

(iii)  $D(E) \subset E$

let  $E = \left\{ 0, 1, \frac{1}{2}, \frac{1}{3}, \dots \right\}$  Ans

$$D(E) = \{0\}$$

$$\Rightarrow D(E) \subset E$$

(iv)  $E = D(E)$  holds for  $E = [a, b]$  Ans  
 $D(E) = [a, b]$   
 $\Rightarrow E = D(E)$

Closed Set: A subset  $E$  of  $\mathbb{R}$  is said to be closed if it contains all its limit points.

or  
 $D(E) \subseteq E$

- Eg
- (i) The set of  $\mathbb{R}$  is closed.
  - (ii) Any  $[a, b]$  is a closed set.
  - (iii) Semi-open interval  $[a, b)$  is neither open nor closed set.
  - (iv) Every finite set  $\{1, 2, 3\}$  is closed.
  - (v) Null set  $\phi$  is both closed and open.

Q Give an example of each of the following

(i) A closed set which is not an interval.  
 $E = [2, 3] \cup [4, 5]$   
 or any finite set

(ii) An interval which is not a closed set  
 Open Interval  $(0, 1)$   
 Semi-open Interval  $[0, 1)$

(iii) A set which is neither an interval nor a closed set.  
 $E = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \dots \right\}$  or  $E = (2, 3) \cup (4, 5)$

(iv) A set which is neither closed nor open.  
Semi-open Interval  $(0, 1]$   
 $E = \left\{ \frac{1}{2}, \frac{2}{3}, \dots, \frac{n}{n+1} \right\}$

(v) A set which is both open & closed.  
 $\emptyset$