Q. Show that the union of an arbitrary collection of closed sets may or may not be closed.

~~let $\{E_i : i \in I\}$ where I is any index set, be an arbitrary collection of closed sets.~~

For each $n \in N$, let $E_n = \left[1, \frac{n+1}{n}\right]$ then each E_n is a closed set.

Further,

$$\bigcup_{n=1}^{\infty} E_n = [1,2] \cup \left[1, \frac{3}{2}\right] \cup \left[1, \frac{4}{3}\right] \cup \ldots$$

$$\bigcup_{n=1}^{\infty} E_n = [1,2], \text{ which is closed.}$$

Now,

for each $n \in Z$, let $E_n = \left[0, \frac{n}{n+1}\right]$, then each E_n is a closed set.

Further,

$$\bigcup_{n=1}^{\infty} E_n = \left[0, \frac{1}{2}\right] \cup \left[0, \frac{2}{3}\right] \cup \left[0, \frac{3}{4}\right] \cup \ldots$$

$$\bigcup_{n=1}^{\infty} E_n = [0,1), \text{ which is not closed.}$$

Therefore, arbitrary union of closed sets may or may not be closed.

Closure of a Set: let E be a subset of R. The closure of set E is the intersection of all closed ~~sets~~ supersets of E. It is denoted by $\overline{E}$. or

$\overline{E}$ is the smallest closed set containing E.

or

$\boxed{\overline{E} = \cap\{F : F \text{ is closed}, E \subseteq F\}}$ $\overline{R} = R$, $\boxed{E \subseteq \overline{E}}$, $\overline{\phi} = \phi$

Perfect Set: A set 'A' is said to be perfect if — $\boxed{A = D(A)}$

Eg → Null set, ~~Any finite set~~
Any closed set.

Ex → ① If $E = (a, b)$ Find → $\overline{E}$

Ans → $\overline{E} = [a, b]$

② If $E = \{1, 2, 3, 4\}$. Find → $\overline{E}$

Ans → $\overline{E} = E = \{1, 2, 3, 4\}$

③ If $E = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$ Find → $\overline{E}$

Ans → $\overline{E} = \left\{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\right\} = E \cup D(E)$

Q. Is subset of a closed set is closed?

Ans → No

Q. Is superset of a closed set is closed?

Ans → No