Imp

Theorem:- If E is any subset of $\mathbb{R}$, then

i) $\bar{E}$ is closed

ii) $\bar{E}$ is the smallest closed superset of E.

iii) E is closed iff $E = \bar{E}$

Proof:- i) We know that intersection of arbitrary collection of closed sets is closed. Since $\bar{E}$ is defined as intersection of certain closed sets (which contain E), it follows that $\bar{E}$ is closed.

ii) Since $\bar{E}$ equals the intersection of all closed superset of E. it is contained in each closed superset of E. Now, since $\bar{E}$ is closed it follows that $\bar{E}$ is the smallest closed superset of E.

iii) Let $E = \bar{E}$, Since $\bar{E}$ is closed. Hence E is also closed.

Conversely

Let E is closed then it is the smallest closed set containing E. Hence $\bar{\bar{E}} = \bar{E}$