Page

is closed. We now show that $\bar{E} = E \cup D(E)$.

Since $E \cup D(E)$ is closed ~~witho~~ superset of E & $\bar{E}$ is the smallest closed superset of E,

$\Rightarrow \quad \bar{E} \subseteq E \cup D(E)$ —— (i)

Next, since $\bar{E}$ is closed, we have $D(\bar{E}) \subseteq \bar{E}$.

Now $\quad E \subseteq \bar{E} \Rightarrow D(E) \subseteq D(\bar{E}) \subseteq \bar{E}$

$\Rightarrow D(E) \subseteq \bar{E}$

since $E \subseteq \bar{E}$ & $D(E) \subseteq \bar{E}$ it follows that

$E \cup D(E) \subseteq \bar{E}$ — (ii)

from (i) & (ii)

$\bar{E} = E \cup D(E)$

13/9 Ex) Find the closure of (a, b)

Sol^n: Let $E = (a, b)$

$\bar{E} = E \cup D(E)$

$D(E) = [a, b]$

$\bar{E} = (a, b) \cup [a, b]$

$\bar{E} = [a, b]$

Ex) Find the closure of $\{1, 2, 3, 4\}$

Let $E = \{1, 2, 3, 4\}$

$D(E) = \phi$

$\bar{E} = E \cup D(E)$

$\bar{E} = \{1, 2, 3, 4\} \cup \phi$

$\bar{E} = \{1, 2, 3, 4\}$

Ex) Find the closure of set of integers $\mathbb{Z}$

$\bar{E} = \mathbb{Z}$ (Ans)

$D(E) = \phi$

Ex) $\{1, 1/2, 1/3 \dots\}$

$D(E) = \{0\}$

$\bar{E} = \{0, 1, 1/2, 1/3, \dots\}$