Real Numbers

Axioms of Real Numbers :-

The system of real numbers can be described by certain means of axioms which can be divided into 3 categories —

(I) **Field Axioms :-** On the set R of real numbers there are two operations denoted by '+' and '·' called addition and multiplication respectively. These operations satisfy the following axioms —

Addition Axioms ⇒

(A_1) Closure law → $\forall\, a, b \in R \Rightarrow (a+b) \in R$

(A_2) Associative law → $a+(b+c) = (a+b)+c \quad \forall\, a, b, c \in R$

(A_3) Existence of Additive Identity →
The real no. '0' called the additive identity satisfies → $a+0 = 0+a = a \quad \forall\, a \in R$

(A_4) Existence of Additive Inverse →
To each element $a \in R$, $\exists$ an element '$-a$' called additive inverse of 'a' such that —
$$a+(-a) = 0 = (-a)+a$$

(A_5) Commutative law → $a+b = b+a \quad \forall\, a, b \in R$

Multiplication Axioms:-

(M1) Closure law → $\forall\, a, b \in R \Rightarrow a \cdot b \in R$

(M2) Associative law → $\forall\, a, b, c \in R \Rightarrow a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(M3) Existence of Multiplicative Identity →
The real number '1' called the multiplicative identity satisfies → $a \cdot 1 = a = 1 \cdot a \;\forall\, a \in R$

(M4) Existence of Multiplicative Inverse →
To each element $a \in R$ $\exists$ an element $\frac{1}{a}$ called multiplicative inverse of 'a', s.t.
$a \cdot \left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right) \cdot a \;\forall\, a \in R$ or $a \cdot a^{-1} = 1 = a^{-1} \cdot a$

(M5) Commutative law → $\forall\, a, b \in R \Rightarrow a \cdot b = b \cdot a$

Distributive laws → $\forall\, a, b, c \in R$:

(D1) $a \cdot (b + c) = a \cdot b + a \cdot c$ (LHDL)

(D2) $(b + c) \cdot a = (b \cdot a) + c \cdot a$ (RHDL)

In view of above axioms, the set R is called a field.