Theorem :- If A & B are two subsets of R then.

i) $A \subset B \Rightarrow \bar{A} \subseteq \bar{B}$

ii) $\overline{A \cup B} = \bar{A} \cup \bar{B}$

iii) $\overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$

iv) $(\bar{\bar{A}}) = \bar{A}$

Pf:- i) $A \subset B$

$\Rightarrow D(A) \subseteq D(B)$

$\Rightarrow A \cup D(A) \subseteq B \cup D(B)$

$\Rightarrow \bar{A} \subseteq \bar{B}$

ii) $\overline{A \cup B} = (A \cup B) \cup D(A \cup B)$

$= (A \cup B) \cup (D(A) \cup D(B))$

$= [A \cup D(A)] \cup [B \cup D(B)]$

$= \bar{A} \cup \bar{B}$

iii) $\cancel{\overline{A \cap B} =}$ $A \cap B \subseteq A \Rightarrow \overline{A \cap B} \subseteq \bar{A}$

$A \cap B \subseteq B \Rightarrow \overline{A \cap B} \subseteq \bar{B}$

Hence $\overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$

iv) we have

if E is closed then $\bar{E} = E$

since $\bar{A}$ is closed

$\therefore (\bar{\bar{A}}) = \bar{A}$

Assignment - I

Q1) Prove that the derived set of a set is a closed set.

Q2) A subset F of R is closed iff it's compliment is open.

Q3) Union of two open sets is open.

Q4) Find the lt point of the set $\left\{\frac{1}{3}, \frac{4}{11}, \frac{3}{8}, \ldots, \frac{2n}{5n+1}\right\}$

Q5) Find the derived set of $\{1+3^{-n} : n \in N\}$