Sequence of Real Numbers

Sequence:- A sequence is a function (or mapping) 'f' from N to R

i.e. $f: N \to R$.

It is denoted by $\langle f(n) \rangle$ or $\{x_n\}$ or $\langle x_n \rangle, n \in N$

$$f(1) = x_1$$
$$f(2) = x_2$$
$$\vdots \qquad \vdots$$
$$f(n) = x_n$$

The numbers $x_1, x_2, x_3, \ldots x_n$ are called the terms of the sequence.

$\langle x_n \rangle = \langle x_1, x_2, x_3, \ldots \rangle$
$\langle y_n \rangle = \langle y_1, y_2, y_3 \ldots \rangle$

Two sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ are said to be equal if —

$$x_n = y_n \quad \forall\ n \in N$$

Eg- ① $\langle n \rangle = \langle 1, 2, 3, \ldots \rangle$

② $\left\langle \frac{1}{n} \right\rangle = \left\langle 1, \frac{1}{2}, \frac{1}{3}, \ldots \right\rangle$

③ $\langle n^2 \rangle = \langle 1, 2^2, 3^2, \ldots \rangle$

④ $\langle 2 \rangle = \langle 2, 2, 2, \ldots \rangle$

⑤ $\langle K \rangle = \langle K, K, K, \ldots \rangle$ (Constant seq.)

⑥ $\langle (-1)^n \rangle = \langle -1, 1, -1, 1, \ldots \rangle$

⑦ $\left\langle \frac{1}{2^n} \right\rangle = \left\langle \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \right\rangle$

⑧ $\langle (-1)^{n-1} \rangle = \langle 1, -1, 1, -1, \ldots \ldots \rangle$

⑨ $\left\langle \frac{n}{n+1} \right\rangle = \left\langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \ldots \right\rangle$

⑩ $\langle -n \rangle = \langle -1, -2, -3, \ldots \ldots \rangle$

Bounded Sequence:- A sequence of $\langle x_n \rangle$ is said to be bounded below if $\exists$ a real no. 'm' such that—

$$x_n \geq m, \quad \forall n \in N$$

In this case 'm' is called lower bound of the sequence $\langle x_n \rangle$.

A sequence of $\langle x_n \rangle$ is said to be bounded above if $\exists$ a real no. 'M' s.t—

$$x_n \leq M, \quad n \in N$$

In this case 'M' is called upper bound of the sequence $\langle x_n \rangle$.

A sequence of $\langle x_n \rangle$ is said to be bounded if it is bounded below as well as bounded above i.e. —

$$\boxed{m \leq x_n \leq M \quad \forall n \in N}$$

or

$$\boxed{|x_n| \leq M \quad \forall n \in N}$$