17/9/22

Monotonic Sequences:- A sequence $\langle x_n \rangle$ is said to be $\longrightarrow$

(i) non-decreasing (or increasing) iff $\longrightarrow$
$x_n \leq x_{n+1}$ $\forall$ $n \in \mathbb{N}$ i.e. $x_1 \leq x_2 \leq x_3 \leq \ldots$

(ii) non-increasing (or decreasing) iff $\longrightarrow$
$x_n \geq x_{n+1}$ $\forall$ $n \in \mathbb{N}$ i.e. $x_1 \geq x_2 \geq x_3 \geq \ldots$

(iii) strictly decreasing iff $\longrightarrow$ $x_n > x_{n+1}$ $\forall$ $n \in \mathbb{N}$
i.e. $x_1 > x_2 > x_3 \ldots$

(iv) strictly increasing iff $\longrightarrow$ $x_n < x_{n+1}$ $\forall$ $n \in \mathbb{N}$
i.e. $x_1 < x_2 < x_3 < \ldots$

Eg- (i) $\langle [n^{1/2}] \rangle = \langle 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3 \ldots \rangle$
is non-decreasing sequence.

(ii) $\langle f(n) \rangle$, where $f(n) = \begin{cases} \frac{2}{n+1}, & n \text{ is odd} \\ \frac{2}{n}, & n \text{ is even} \end{cases}$

$= \langle 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \ldots \rangle$

is non-increasing or decreasing sequence.

(iii) $\left\langle \frac{n}{n+1} \right\rangle = \left\langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots \right\rangle$

is strictly increasing sequence.

(iv) $\langle a^n \rangle, a > 1 = \langle a, a^2, a^3, \ldots \rangle$
is strictly increasing sequence.

(v) $\left\langle \frac{1}{n} \right\rangle = \left\langle 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \right\rangle$
is strictly decreasing sequence.

(vi) $\langle a^n \rangle$, provided $0 < a < 1$
is strictly decreasing sequence.

(vii) $\left\langle 1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4}, \ldots \right\rangle$
is not a monotonic sequence.

(viii) $\langle 1, -1, 1, -1, \ldots \rangle$
is not a monotonic sequence.

Q. Is the sequence $\left\langle \frac{2^n}{n!} \right\rangle$ monotonic non-increasing or non-decreasing? Find bounds of this sequence if any.

$$\left\langle \frac{2^n}{n!} \right\rangle = \left\langle 2, \frac{2^2}{2!}, \frac{2^3}{3!}, \frac{2^4}{4!} \ldots \right\rangle$$

$$= \left\langle 2, 2, \frac{4}{3}, \frac{2}{3}, \ldots \right\rangle$$

is monotonic non-increasing sequence
Upper Bound = 2
Lower Bound = 0