19/9/22

Q Is the sequence $\langle f(n) \rangle$ where $x_n = f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ a monotonic sequence?

We have —

$$x_{n+1} - x_n = \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \frac{1}{n+1}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

$$x_{n+1} - x_n = \frac{1}{n+1} > 0 \quad \forall\ n \in \mathbb{N}$$

$$x_{n+1} - x_n > 0 \quad \forall\ n \in \mathbb{N}$$

$$x_{n+1} > x_n$$

Hence, the given sequence is monotonically strictly increasing sequence.

Q Find the bounds of the sequence $\langle x_n \rangle$ where $x_n = \frac{4n-1}{5n+2}$

We have,

$$x_{n+1} - x_n = \frac{4n+3}{5n+7} - \left(\frac{4n-1}{5n+2}\right)$$

$$= \frac{20n^2 + 23n + 6 - 20n^2 - 23n + 7}{(5n+7)(5n+2)}$$

$$x_{n+1} - x_n = \frac{13}{(5n+7)(5n+2)} > 0 \quad \forall\ n \in \mathbb{N}$$

$$x_n < x_{n+1} \quad \forall\ n \in \mathbb{N}$$

Lower Bound $= x_1 = \frac{3}{7}$ Ans

Upper Bound $= \lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{4n-1}{5n+2} = \frac{4}{5}$ Ans

Convergent Sequence: A sequence of $\langle x_n \rangle$ is said to be convergent if there is a real number 'l' such that for each positive number ϵ, however small, $\exists$ a positive integer 'm' in such a way that —

$$|x_n - l| < \epsilon \quad \forall \quad n \geq m$$

or

$$\lim_{n\to\infty} x_n = l \text{ (Finite real no.)}$$

or

$$x_n \to l$$

$$l - \epsilon < x_n < l + \epsilon$$

$$x_n \in (l-\epsilon, l+\epsilon)$$

Eg ① $\left\langle \frac{1}{n} \right\rangle$ $\lim_{n\to\infty} \frac{1}{n} = 0 \Rightarrow$ Convergent Sequence

② $\left\langle \frac{n}{n+1} \right\rangle$ $\lim_{n\to\infty} \frac{n}{n+1} = 1 \Rightarrow$ Convergent sequence

③ $\langle k \rangle$ $\lim_{n\to\infty} k = k \Rightarrow$ Convergent sequence

④ $\langle 2^{-n} \rangle$ $\lim_{n\to\infty} 2^{-n} = 0 \Rightarrow$ Convergent sequence

⑤ $\left\langle \frac{(-1)^n}{n} \right\rangle$ $\lim_{n\to\infty} \frac{(-1)^n}{n} = 0 \Rightarrow$ Convergent sequence

⑥ $\langle (-1)^{n-1} \rangle$ Not convergent. Limit is not unique.

⑦ $\langle n^2 \rangle$ $\lim n^2 = \infty \Rightarrow$ Divergent

⑧ $\langle n^2(-1)^n \rangle$ Oscillates b/w ∞ and $-\infty$.