Q. Show that the sequence $\left\langle \frac{n}{n+1} \right\rangle$ converges to 1.

Sol^n → let $\epsilon > 0$

$$\left|\frac{n}{n+1} - 1\right| < \epsilon \Leftrightarrow \left|\frac{1}{n+1}\right| < \epsilon \Rightarrow \frac{1}{n+1} < \epsilon$$

$$\Rightarrow n+1 > \frac{1}{\epsilon} \Rightarrow n > \frac{1-\epsilon}{\epsilon} = \delta \text{ (say)}$$

$$n \geq m \Rightarrow |x_n - l| < \epsilon$$

It should be noted that δ is not necessarily an integer and may be negative also.

Now, let $m = \max\{[\delta]+1, 1\}$ where δ is greatest integer.

Then, $n \geq m \Rightarrow \left|\frac{n}{n+1} - 1\right| < \epsilon$

Hence, the given sequence converges to 1.

Q. Show that the sequence $\left\langle \frac{1}{n^p} \right\rangle$ converges to 0. when $p>0$.

Sol^n let $\epsilon>0$

$$\left|\frac{1}{n^p}-0\right|<\epsilon \Leftrightarrow \frac{1}{n^p}<\epsilon \Rightarrow n^p>\frac{1}{\epsilon} \Rightarrow n>\left(\frac{1}{\epsilon}\right)^{1/p}$$

let $\delta=\left(\frac{1}{\epsilon}\right)^{1/p}$ (say)

Now, let $m=[\delta]+1$, we have —

$$n \geq m \Rightarrow \left|\frac{1}{n^p}-0\right|<\epsilon$$

Hence the given sequence converges to 0.

Q. Show that the sequence $\langle a_n \rangle$ defined by $a_n=\left(1+\frac{1}{n}\right)^n$ is monotonic increasing and convergent. Also show that its limit lies between 2 and 3. Further show that $\lim_{n\to\infty} a_n = e$

Sol$^n \rightarrow$ $a_n=\left(1+\frac{1}{n}\right)^n$

$$a_n = 1+n\cdot\frac{1}{n}+\frac{n(n-1)}{2!}\cdot\frac{1}{n^2}+\cdots+\frac{n(n-1)\cdots 1}{n!}\cdot\frac{1}{n^n}$$

$$a_n = 1+1+\frac{1}{2!}\left(1-\frac{1}{n}\right)+\frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)+\cdots+\frac{1}{n!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{n-1}{n}\right)$$