Q. Prove that the set R^+ of positive real numbers is bounded below but not bounded above.

Since every negative number or zero is a lower bound of R^+ we find that R^+ is bounded below.

Let if possible K be an upper bound of R^+.
Since $1 \in R^+$ and 'K' is an upper bound of R^+.
We have $K \geq 1$ which means $K > 0$
Therefore $K+1 > 0$ and $K+1 \in R^+$

Since, $K+1 > K$
K cannot be upper bound of R^+.
Thus, R^+ is not bounded above.

——— Proved

Q. Find the glb and lub of the set
$S = \{x \in Z : x^2 \leq 25\}$
$S = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

glb $S = -5$ Ans
lub $S = 5$ Ans

Q. Find the supremum and infimum of the set $S = \{(-1)^n n : n \in N\}$ if they exist

$S = \{(-1)^n n : n \in N\}$

$S = \{-1, 2, -3, 4, -5, 6, \ldots\}$

$S = \{\ldots, -5, -3, -1, 2, 4, 6, \ldots\}$

From above, S is neither bounded above nor bounded below.

Hence, Supremum and Infimum of S does not exist.

Q. Find the supremum and infimum of $S = \left\{1 + \frac{(-1)^n}{n} : n \in N\right\}$ if they exist.

$S = \left\{0, \frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}, \frac{7}{6}, \frac{6}{7}, \frac{9}{8}, \frac{8}{9}, \ldots\right\}$

$S = \left\{0, \frac{2}{3}, \frac{4}{5}, \ldots\right\} \cup \left\{\frac{3}{2}, \frac{5}{4}, \ldots\right\}$

$S = \left\{0, \frac{2}{3}, \frac{4}{5}, \ldots \frac{2n}{2n+1}, \ldots\right\} \cup \left\{\frac{3}{2}, \frac{5}{4}, \ldots \frac{2n+1}{2n}, \ldots\right\}$

Evidently, 0 is the smallest element in S and $\frac{3}{2}$ is the greatest element in S.

Hence, $\text{Inf } S = 0$

$\text{Sup } S = \frac{3}{2}$