3/8/22 property.

Lecture-3

Commutative Group/ Abelian Group:-

A group $(G, *)$ is commutative group if $a*b = b*a \ \forall \ a, b \in G$

Order of a Group:-

Let $(G, *)$ is a group. The order of group G denoted as $|G|$ or $O(G)$ is the number of elements in non-empty set G.

Ex→ ① $(Z, +)$ is an infinite group

Order of $Z = |Z| = $ Infinite.

② $G = \{1, -1, i, -i\}$ $(G, \cdot)$

$O(G) = |G| = 4$

Order of elements in $(G, *)$:-

let $a \in G$. Order of element 'a' in G is the least positive integer 'n' s.t —

$$\boxed{a * a * a * \cdots n \text{ times} = e}$$

- If $* = \cdot$ $\quad \boxed{a^n = e}$
- If $* = +$ $\quad \boxed{na = e}$

Ex → $G = \{1, -1, i, -i\}$ $\quad (G, \cdot)$

$O(G) = 4$

In this case, $e = 1$

$(-1) \cdot (-1) = (-1)^2 = 1$ $\quad\quad (1)^1 = 1$

$O(-1) = 2$ $\quad\quad O(1) = 1$

$(i)^4 = 1$ $\quad\quad (-i)^4 = 1$

$O(i) = 4$ $\quad\quad O(-i) = 4$

Note → If $O(G) = 4$

Order of elements in G can be $= 1/2/4$ i.e. factors of 4.

Self Inverse: let $a \in (G, *)$ then a is called self inverse if —

$\boxed{a * a = e}$

Note → ① If $* = \cdot$ then $\boxed{a^2 = 1}$ i.e. $\boxed{a^{-1} = a}$

② If 'a' is self inverse and a is non-identity then order of 'a' is always '2'.

③ Order of identity element is always 1.

④ An element and its inverse both have same order.

$\boxed{O(a) = O(a^{-1})}$

Order Description Table:

① $G = \{1, -1, i, -i\}$ $(G, \cdot)$ $O(G) = 4$

1/2/4

Element	Order
1	1
-1	2
i	4
-i	4

② $G = \{1, \omega, \omega^2\}$ $(G, \cdot)$ $O(G) = 3$

1/3

Element	Order
1	1
ω	3
ω^2	3