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Q. Find the value of $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}}$

$$= \lim_{n\to\infty}\left(\frac{3}{\sqrt{n}}+2\right)$$

$$= 2 \quad \underline{\text{Ans}}$$

Q. Find the value of $\lim_{n\to\infty} \frac{(3n+1)(n-2)}{n(n+3)}$

$$= \lim_{n\to\infty}\left[\frac{\cancel{n^2}\left(3+\frac{1}{n}\right)\left(1-\frac{2}{n}\right)}{\cancel{n^2}\left(1+\frac{3}{n}\right)}\right]$$

$$= 3 \quad \underline{\text{Ans}}$$

Cauchy's Theorems on Limits:-

Theorem (Cauchy's first Theorem on limits) →

If a sequence $\langle x_n \rangle$ converges to the limit l and $S_n = \frac{x_1 + x_2 + \cdots + x_n}{n}$ then the sequence of $\langle S_n \rangle$ also converges to the same limit 'l'.

Theorem (Cauchy's second Theorem on limits) →

If $\langle a_n \rangle$ be a sequence of positive terms, then → $\lim (a_n)^{1/n} = \lim \frac{a_n}{a_{n-1}} = \lim \frac{a_{n+1}}{a_n}$ provided the later limit exists.

Proof → We have —

$$(a_n)^{1/n} = \left(\frac{a_1}{1} \cdot \frac{a_2}{a_1} \cdot \frac{a_3}{a_2} \cdots \frac{a_{n-1}}{a_{n-2}} \cdot \frac{a_n}{a_{n-1}} \right)^{1/n}$$

On taking log on both sides —

$$\log (a_n)^{1/n} = \frac{1}{n} \left[\log\left(\frac{a_1}{1}\right) + \log\left(\frac{a_2}{a_1}\right) + \cdots + \log\left(\frac{a_n}{a_{n-1}}\right) \right]$$

$$= \frac{x_1 + x_2 + \cdots + x_n}{n} \text{ (say)}$$

By using Cauchy's first theorem on limits, we get ⟶

$$\lim_{n\to\infty} \log (a_n)^{1/n} = \lim_{n\to\infty} x_n = \lim_{n\to\infty} \log\left(\frac{a_n}{a_{n-1}}\right)$$

$$\log \lim_{n\to\infty} (a_n)^{1/n} = \log\left(\lim_{n\to\infty} \frac{a_n}{a_{n-1}} \right)$$

$$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \frac{a_n}{a_{n-1}}$$

Proved

Cesaro's Theorem :- If the sequences $\langle a_n \rangle$ and $\langle b_n \rangle$ converges to a & b respectively then the sequence $\langle c_n \rangle$ defined by —

$$c_n = \frac{1}{n}(a_1 b_n + a_2 b_{n-1} + \cdots + a_{n-1} b_2 + a_n b_1)$$

converges to ab.

Theorem:- If a sequence of $\langle a_n \rangle$ such that $\frac{a_{n+1}}{a_n} \to l$ and $|l|<1$ then

$$\lim_{n\to\infty} a_n = 0$$

Q. Prove That $\to \lim_{n\to\infty} \frac{1}{n}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right)=0$

$$S_n = \frac{x_1+x_2+\cdots+x_n}{n} \quad \text{(say)}$$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

Hence by Cauchy's first theorem on limits, we get $\longrightarrow$

$$\lim_{n\to\infty} S_n = 0$$

$$\lim_{n\to\infty} \frac{1}{n}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right)=0$$

Proved

Q. Prove That $\to \lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1$

$$= \lim_{n\to\infty} \frac{1}{n}\left(\frac{n}{\sqrt{n^2+1}}+\frac{n}{\sqrt{n^2+2}}+\cdots+\frac{n}{\sqrt{n^2+n}}\right)$$

Now, $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}} = 1$

Hence, by Cauchy's first theorem on limits we get $\longrightarrow$

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right)=1$$

Proved