

Subsequences - let  $\langle x_n \rangle$  be a sequence of real numbers and let  $\langle i_n \rangle$  be a strictly increasing sequence of natural numbers i.e.  $i_1 < i_2 < i_3 < \dots$  then the sequence  $\langle x_{i_n} \rangle = \langle x_{i_1}, x_{i_2}, x_{i_3}, \dots \rangle$  is called a subsequence of  $\langle x_n \rangle$ .

Eg  $\rightarrow$   $\langle x_{2n} \rangle = \langle x_2, x_4, x_6, \dots \rangle$  and  $\langle x_{2n+1} \rangle = \langle x_1, x_3, x_5, \dots \rangle$  are subsequences of  $\langle x_n \rangle = \langle x_1, x_2, x_3, x_4, \dots \rangle$

$\star$   $\langle \frac{1}{n} \rangle = \langle 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \rangle$

$\langle \frac{1}{n+2} \rangle = \langle \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots \rangle$

$\langle \frac{1}{2n-1} \rangle = \langle 1, \frac{1}{3}, \frac{1}{5}, \dots \rangle$

$\langle \frac{1}{n^2} \rangle = \langle 1, \frac{1}{4}, \frac{1}{9}, \dots \rangle$

$\Rightarrow \langle \frac{1}{n+2} \rangle, \langle \frac{1}{2n-1} \rangle, \langle \frac{1}{n^2} \rangle$  are subsequences of  $\langle \frac{1}{n} \rangle$

$\#$   $\langle \frac{1}{2}, 1, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \dots \rangle$  and

$\langle 1, 0, \frac{1}{3}, 0, \frac{1}{5}, \dots \rangle$  are not subsequences of  $\langle \frac{1}{n} \rangle$