Lecture-36

1/11/22

Solⁿ

Q (a) $G = Z_{25} \oplus Z_5$ cyclic?

$|Z_{25} \oplus Z_5| = 25 \times 5 = 125$

$Z_{25} \oplus Z_5 = \langle (a,b) \rangle$

$(a,b) \in Z_{25} \oplus Z_5$

$|(a,b)| = \text{lcm}(|a|, |b|)$

$\|Z_{25}\|$	$\|Z_5\|$
$\|a\|$	$\|b\|$
1	1
5	5
25	

$\text{lcm} = 1 = \text{lcm}(1,1)$

$5 = \text{lcm}(1,5)$

$5 = \text{lcm}(5,5)$

$5 = \text{lcm}(5,1)$

$25 = \text{lcm}(25,1)$

$25 = \text{lcm}(25,5)$

So $|(a,b)| = 1$ or 5 or 25

$\Rightarrow |Z_{25} \oplus Z_5| \neq |(a,b)|$

$\Rightarrow Z_{25} \oplus Z_5$ is not a cyclic group

(b) Determine ~~all~~ the no. of elements of order 5 in $G = Z_{25} \oplus Z_5$

Case I $\rightarrow$ $|a| = 1$, $|b| = 5$, $|(a,b)| = 5$

Case II $\rightarrow$ $|a| = 5$, $|b| = 1$, $|(a,b)| = 5$

Case III $\rightarrow$ $|a| = 5$, $|b| = 5$, $|(a,b)| = 5$

Case I $\rightarrow$ $\phi(1) \cdot \phi(5) = 1 \times 4 = 4$

Case II $\rightarrow$ $\phi(5) \cdot \phi(1) = 4 \times 1 = 4$

Case III $\rightarrow$ $\phi(5) \cdot \phi(5) = 4 \times 4 = 16$

Total no. of elements of order 5 in $G = Z_{25} \oplus Z_5 = 4 + 4 + 16 = 24$ Ans

Q (c) Determine ~~all~~ the no. of elements of order 25 in $G = Z_{25} \oplus Z_5$

Case I → $|a| = 25$, $|b| = 1$, $|(a,b)| = 25$

$$\phi(25) \cdot \phi(1) = 5^2 - 5^1 = 20$$

Case II → $|a| = 25$, $|b| = 5$, $|(a,b)| = 25$

$$\phi(25) \cdot \phi(5) = 20 \times 4 = 80$$

Total no. of elements of order 25 in $G = Z_{25} \oplus Z_5 = 20 + 80 = 100$ Ans

Q Determine the no. of cyclic subgroups of order 10 in $Z_{100} \oplus Z_{25}$

let H is cyclic subgroup of $Z_{100} \oplus Z_{25}$

$|H| = 10$

$|H| = |\langle (a,b) \rangle| = 10$

$(Z_{100}) = |a| = 1, 2, 4, 5, 10, 20, 25, 50, 100$

$(Z_{25}) = |b| = 1, 5, 25$

Case I → $\text{lcm}(10, 1) = 10$

$$\phi(10) \cdot \phi(1) = 10\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right) = 4$$

Case II → $\text{lcm}(10, 5) = 10$

$$\phi(10) \cdot \phi(5) = 4 \times 4 = 16$$

Case III → $\text{lcm}(2, 5) = 10$

$$\phi(2) \cdot \phi(5) = 1 \times 4 = 4$$

No. of cyclic subgroups of order 10 in $Z_{100} \oplus Z_{25}$

$= \frac{24}{\phi(10)} = \frac{24}{4} = 6$ Ans

$H_1, H_2, H_3, H_4, H_5, H_6$ (all are distinct)

Q. Determine the no. of cyclic subgroups of order 20 in $Z_{100} \oplus Z_{25}$

Let H be a cyclic subgroup of order 20 in $Z_{100} \oplus Z_{25}$

$|H| = 20$

$|H| = |\langle(a,b)\rangle| = 20$

Case I → $\text{lcm}(20,1) = 20$

$\phi(20)\,\phi(1) = 20\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 8$

Case II → $\text{lcm}(20,5) = 20$

$\phi(20)\cdot\phi(5) = 8\cdot 4 = 32$

Case III → $\text{lcm}(4,5) = 20$

$\phi(4)\cdot\phi(5) = 2\cdot 4 = 8$

Total elements $= 48$

$|H| = 20$

$\phi(20) = 8$

No. of cyclic subgroups in $Z_{100} \oplus Z_{25}$ of order 20

$= \frac{48}{\phi(20)} = \frac{48}{8} = 6$ Ans