(3) Prove That → for any real number x,

$$\lim_{n\to\infty} \frac{x^n}{n!} = 0$$

let $a_n = \frac{x^n}{n!}$

(We know that if $\langle a_n \rangle$ s.t. $\frac{a_{n+1}}{a_n} \to l$ & $|l|<0$

then $\lim_{n\to\infty} a_n = 0$)

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right]$$

$$= \lim_{n\to\infty} \left[\frac{x^n \cdot x}{(n+1)\,n!} \times \frac{n!}{x^n} \right]$$

$$= \lim_{n\to\infty} \frac{x}{n+1}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$$

$\Longrightarrow \langle a_n \rangle$ s.t. $\frac{a_{n+1}}{a_n} \to 0$ where $|0|<1$

$\Rightarrow \lim_{n\to\infty} a_n = 0$

i.e. $\lim_{n\to\infty} \frac{x^n}{n!} = 0$

—— Proved