14/10/22

Q. Test the convergence of the series:-

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots$$

Sol^n Since the given series is alternating and the terms are in decreasing order.

Here, $u_n=\frac{1}{n\sqrt{n}}$

$\lim u_n = 0$

Hence, by leibnitz Test, the given series is convergent.

Q. Test the convergence of the series —

$$1-\frac{1}{2}+\frac{1\cdot 3}{2\cdot 4}-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}+\ldots$$

Sol^n → Since the given series is alternating and the terms are in decreasing order.

Here, $u_n=\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}$

$$u_n=\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdots\frac{2n-1}{2n}$$

$$u_n=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{6}\right)\cdots\left(1-\frac{1}{2n}\right)$$

$$\log u_n=\log\left(1-\frac{1}{2}\right)+\log\left(1-\frac{1}{4}\right)+\log\left(1-\frac{1}{6}\right)+\cdots+\log\left(1-\frac{1}{2n}\right) \quad \text{——— (1)}$$

$$\log(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\ldots\ldots, \quad |x|<1$$

$$=-x-\left(\frac{x^2}{2}+\frac{x^3}{3}+\ldots\ldots\right)$$

Now, $0<x<1$

$\log(1-x) < -x$ ——— ②

Using eq ② in ①, we get →

$$\log u_n < \frac{-1}{2} - \frac{1}{4} - \frac{1}{6} - \cdots - \frac{1}{2n}$$

$$\log u_n < \frac{-1}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

Since $\sum \frac{1}{n}$ is divergent. Consequently,

$\log u_n \to -\infty$ as $n \to \infty$

$\Rightarrow \lim \log u_n = -\infty$

$\Rightarrow \log(\lim u_n) = -\infty$

$\lim u_n = e^{-\infty}$

$\lim u_n = 0$

Hence, the given series is convergent.

Q Test the convergence of the series —

$$\left(\frac{1}{7} - \frac{1}{\log 7}\right) - \left(\frac{1}{7} - \frac{1}{\log 8}\right) + \left(\frac{1}{7} - \frac{1}{\log 9}\right) - \cdots$$

Solⁿ The given series is alternating and the terms are in non-decreasing order.

Here, $u_n = \frac{1}{7} - \frac{1}{\log(n+6)}$

$$\lim u_n = \frac{1}{7} - \frac{1}{\infty} = \frac{1}{7} \neq 0$$

Hence, the given series is oscillatory.