28/10/22

D'Alembert's Ratio Test (Ratio Test):-

☆ $\sum u_n$ is convergent if $\lim \frac{u_n}{u_{n+1}} > 1$

☆ $\sum u_n$ is divergent if $\lim \frac{u_n}{u_{n+1}} < 1$

☆ If $\lim \frac{u_n}{u_{n+1}} = 1$ then test fails.

Q. ~~If~~ Is the infinite series —

$\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \ldots$ convergent or divergent?

$$u_n = \frac{x^n}{n(n+1)}$$

$$\frac{u_n}{u_{n+1}} = \frac{x^n}{n\cancel{(n+1)}} \times \frac{\cancel{(n+1)}(n+2)}{x^{n+1}}$$

$$\lim \frac{u_n}{u_{n+1}} = \frac{1}{x} \lim \left(1 + \frac{2}{n}\right)$$

$$\lim \frac{u_n}{u_{n+1}} = \frac{1}{x}$$

☆ If $\frac{1}{x} > 1$ or $x > 1$ then the given series is convergent,

☆ If $\frac{1}{x} < 1$ or $x < 1$, then the given series is divergent.

If $\frac{1}{x} = 1$ or $x = 1$, then test fails

At $x=1$

$$S = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots$$

$$S = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \ldots$$

$$S = 1$$

$\Rightarrow \lim S_n = S = 1$ (finite quantity)

Hence, the given series is convergent at $x=1$.

Q Is the infinite series —

$$\frac{x}{1\cdot 2} + \frac{x^2}{3\cdot 4} + \frac{x^3}{5\cdot 6} + \frac{x^4}{7\cdot 8} + \ldots \quad \text{at } x>0$$

convergent or divergent?