9/10/22

Q. Test the nature of the series

$2+\frac{3x}{2}+\frac{4x^2}{3}+\frac{5x^3}{4}+\ldots\ldots$ at $x>0$

$U_n = \frac{n+1}{n} x^{n-1}$

$U_{n+1} = \frac{n+2}{n+1} x^n$

$\frac{U_n}{U_{n+1}} = \frac{(n+1)^2 \cdot x^{n-1}}{n \cdot (n+2)\ x^n}$

$\lim \frac{U_n}{U_{n+1}} = \frac{1}{x} \frac{\left(1+\frac{1}{n}\right)^2}{\left(1+\frac{2}{n}\right)}$

$\lim \frac{U_n}{U_{n+1}} = \frac{1}{x}$

If $\frac{1}{x}>1$ or $x<1$ then the given series is convergent.

If $\frac{1}{x}<1$ or $x>1$, then the given series is divergent.

At $x=1$, test fails

$S = 2+\frac{3}{2}+\frac{4}{5}+\frac{5}{4}+\ldots$

$U_n = \frac{n+1}{n}$

$\lim U_n = 1 \neq 0$

$\Rightarrow$ the given series is divergent at $x=1$.

Q Test the nature of the series :-

$$1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\cdots$$

$$U_n=\frac{n^p}{n!}$$

$$U_{n+1}=\frac{(n+1)^p}{(n+1)!}$$

$$\lim\frac{U_n}{U_{n+1}}=\lim\frac{n^p\,(n+1)\,\cancel{n!}}{(n+1)^p\,\cancel{n!}}$$

$$=\lim\frac{1}{\left(1+\frac{1}{n}\right)^p}(n+1)$$

$$\lim\frac{U_n}{U_{n+1}}=\infty>1$$

Hence, the given series is convergent for all p.

Q. Test the nature of the series —

$$x+\frac{3}{5}x^2+\frac{8}{10}x^3+\frac{15}{17}x^4+\cdots+\frac{n^2-1}{n^2+1}x^n+\cdots,\ x>0$$

$$U_n=\frac{n^2-1}{n^2+1}x^n$$

$$U_{n+1}=\frac{(n+1)^2-1}{(n+1)^2+1}x^{n+1}$$

$$=\frac{(n+2)n}{n^2+2n+2}x^{n+1}$$

$$\frac{U_n}{U_{n+1}}=\frac{n^2-1}{n^2+1}\frac{x^n}{x^{n+1}}\left(\frac{n^2+2n+2}{n(n+2)}\right)x$$

$$\lim \frac{u_n}{u_{n+1}} = \lim \frac{1}{x} \frac{\left(1-\frac{1}{n^2}\right)\left(1+\frac{2}{n}+\frac{2}{n^2}\right)}{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2}{n}\right)}$$

$$= \frac{1}{x}$$

If $\frac{1}{x} > 1$ or $x < 1$, then series is convergent.

If $\frac{1}{x} < 1$ or $x > 1$ then series is divergent.

At $x = 1$ test fails

$$\lim u_n = \lim \frac{n^2-1}{n^2+1}$$

$\lim u_n = 1 \neq 0$

Hence the given series is divergent at $x = 1$.