Q) $x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \cdots \frac{n^2-1}{n^2+1}x^n + \cdots,\ x>0$

$$u_n = \frac{n^2-1}{n^2+1} \cdot x^n$$

$$u_{n+1} = \frac{(n+1)^2-1}{(n+1)^2+1} \cdot x^{n+1} = \frac{n^2+2n+1-1}{n^2+2n+2} \cdot x^n \cdot x$$

$$\frac{u_n}{u_{n+1}} = \frac{\frac{n^2-1}{n^2+1} \cdot x^n}{\frac{n^2+2n}{n^2+2n+2} \cdot x^n \cdot x} = \frac{n^2-1 \cdot x^n \cdot (n^2+2n+2)}{x^n \cdot x \cdot n(n+2)(n^2+1)}$$

$$= \frac{(n^2-1)(n^2+2n+2)}{x \cdot n(n+2)(n^2+1)}$$

$$\frac{(1-1/n^2)(1+2/n+2/n^2)}{n(1+2/n)(1+1/n^2)}$$

$$= \frac{1}{x} \quad (1-1$$

$$= 1/x$$

At $x = 1$

$$1 + \frac{3}{5} + \frac{8}{10} + \frac{15}{17} + \cdots + \frac{n^2-1}{n^2+1}$$

$$\lim u_n = \left(\frac{1-1/n^2}{1+1/n^2}\right) = 1$$

$\Rightarrow$ div

Q) $\sum \frac{a^n}{a^n+x^n}$, where $a \neq 0$

$$u_n = \frac{a^n}{a^n+x^n}, \quad u_{n+1} = \frac{a^{n+1}}{a^{n+1}+x^{n+1}}$$

Date 29/10
Page

$$\frac{u_n}{u_{n+1}} = \frac{\dfrac{a^n}{a^n+x^n}}{\dfrac{a^{n+1}}{a^{n+1}+x^{n+1}}} = \frac{a^n\cdot(a^{n+1}+x^{n+1})}{(a^n+x^n)\cdot \cancel{a^n}\cdot a}$$

$$= \frac{a^{n+1}+x^{n+1}}{(a^n+x^n)\cdot a}$$

next step

$$= \frac{a^n\cdot a + x^n\cdot x}{(a^n\cdot \cancel{a} + x^n\cdot \cancel{x})a}$$

$$= \frac{a^{n+1} + x^n\cdot x}{a^{n+1} + a\cdot x^n}$$

$$\lim \frac{1}{a}\,\frac{x^{n+1}}{x^n}\,\frac{[1+(a/x)^{n+1}]}{[1+(a/x)^n]} = 1/a\left(\frac{a^n\cdot a + x^n\cdot x}{(a^n+x^n)}\right)$$

If $x>a$

$= x/a > 1$ (conv)

, If $x=a$

$$\lim u_n = \lim\left(\frac{1}{1+(x/a)^n}\right)$$

$= 1/2 \neq 0$

If $x<1$

$$\lim u_n = \lim\left(\frac{1}{1+(x/a)^n}\right)$$

$= 1$

$\rightarrow$ $\left(\frac{x}{a}\right)^n$ let's say

$\lim 1/2^n = 0$