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→ Cauchy's n^{th} root test (or Root Test):-
An infinite series Σu_n of the terms is convergent if $\lim (u_n)^{1/n} < 1$, divergent if $\lim (u_n)^{1/n} > 1$
$= 1$ (test fails).

Ques Test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \text{---}$$

$$u_n = \frac{x^{n-1}}{n^{n-1}} \quad \frac{x^n}{(n+1)^n}$$

(Neglecting Ist term)

$$(u_n)^{1/n} = \frac{x}{(n+1)}$$

$$\lim (u_n)^{1/n} = \frac{x}{\infty} = \boxed{0 < 1}$$

Hence, convergent series for all values of x

Ques Test the convergence of series:-

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \text{---}$$

$$u_n = \left[\frac{(n+1)^{n+1}}{n^{n+1}} - \frac{n+1}{n}\right]^{-n}$$

$$(u_n)^{1/n} \quad \left(\frac{(n+1)^n}{n^n}\frac{(n+1)}{n} - \frac{n+1}{n}\right)^{-n \times \frac{1}{n}}$$

$$= \left[\frac{(n+1)^{n+1}}{n^{n+1}} - \frac{n+1}{n}\right]^{-1}$$

$$\Rightarrow \frac{n^{n+1}}{(n+1)^{n+1}} - \frac{n}{n+1} \quad \left[\left(\frac{n+1}{n}\right)^{n+1} - \left(\frac{n+1}{n}\right)\right]^{-1}$$

$$\Rightarrow \frac{n^{n+1} - n(n+1)^n}{(n+1)^{n+1}} =$$

$$\Rightarrow (u_n)^{1/n} = \left[\left(1+\frac{1}{n}\right)^{n+1} - \left(1+\frac{1}{n}\right)\right]^{-1}$$

$$\Rightarrow \left[\left(1+\frac{1}{n}\right)^{n}\left(1+\frac{1}{n}\right) - \left(1+\frac{1}{n}\right)\right]^{-1}$$

$$\Rightarrow [e\cdot 1 - 1]^{-1}$$

$$= (e-1)^{-1}$$

$$\frac{1}{e-1} < 1$$

Hence, it is convergent series.

→ Comparison Test :- If all the terms of a series Σu_n are (+ve).
Σv_n (let), for which $\lim \frac{u_n}{v_n}$ = non zero finite quantity
then, the nature of both series are identical.

Ques Test the nature of the series :-

$$\frac{1}{2} + \frac{\sqrt{2}}{5} + \frac{\sqrt{3}}{10} + \text{-----} + \frac{\sqrt{n}}{n^2+1}.$$

$$u_n = \frac{\sqrt{n}}{n^2+1}$$

$$\Rightarrow \frac{1}{n^{-1/2}\cdot n^2\left(1+\frac{1}{n^2}\right)} = \frac{1}{n^{3/2}\left(1+\frac{1}{n^2}\right)}$$

$$V_n = \Sigma \frac{1}{n^{3/2}}$$

$$\lim \frac{u_n}{v_n} = \frac{\frac{1}{\cancel{n^{3/2}}\left(1+\frac{1}{n^2}\right)}}{\frac{1}{\cancel{n^{3/2}}}} \Rightarrow \frac{1}{\left(1+\frac{1}{n^2}\right)} = \boxed{1}$$

Now $\frac{3}{2} > 1$, hence converg

Since $\lim \frac{U_n}{V_n}$ is a non-zero finite quantity, therefore nature of given series identical with the series $\sum \frac{1}{n^{3/2}}$. But $\sum \frac{1}{n^{3/2}}$ is convergent by p-series test. Hence, by comparison test, the given series is convergent.

Ques:- Test the convergence of series whose n^{th} term is given by $U_n = \sqrt{n^2+1} - n$.

$$U_n = n\sqrt{1+\frac{1}{n^2}} - n.$$

$$= n\left[\sqrt{1+\frac{1}{n^2}} - 1\right]$$

$$= n\left[\left(1+\frac{1}{n^2}\right)^{1/2} - 1\right]$$

$$= n\left[\cancel{1} + \frac{1}{2n^2} - \frac{1}{8n^4} + \cdots - \cancel{1}\right]$$

$$= \frac{1}{n}\left[\frac{1}{2} - \frac{1}{8n^2} + \cdots\right]$$

$V_n = \frac{1}{n}$ $\boxed{p = 1}$. $\left[\begin{matrix} p > 1 \text{ conv} \\ p \leq 1 \text{ dive} \end{matrix}\right]$.

$\lim \frac{U_n}{V_n} = \frac{1}{2}$ (non-zero).

$\sum V_n$ is divergent, then $\sum U_n$ also divergent.