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Ex:- Find whether the following series is convergent or divergent.

$$1 + a + \frac{a(a+1)}{1\cdot 2} + \frac{a(a+1)(a+2)}{1\cdot 2\cdot 3} + \cdots$$

Soln:-

$$u_n = \frac{a(a+1)(a+2)\cdots(a+n-2)}{1\cdot 2\cdot 3\cdots n-1}$$

$$u_n = \frac{a(a+1)\cdots(a+n-2)}{(n-1)!}$$

$$u_{n+1} = \frac{a(a+1)\cdots(a+n-1)}{n!}$$

$$\frac{u_n}{u_{n+1}} = \frac{a(a+1)\cdots(a+n-2)\times n\cancel{(n-1)!}}{\cancel{(n-1)!}\; a(a+1)\cdots\underset{(a+n-2)}{\cdot}(a+n-1)}$$

$$\frac{u_n}{u_{n+1}} = \frac{n}{a+n-1} = \frac{1}{\left(1-\frac{1-a}{n}\right)}$$

$$\lim u_n/u_{n+1} = 1$$

$$= \lim\left[n\left(\frac{n}{a+n-1} - 1\right)\right] = \lim n\left(\frac{1-a}{n-(1-a)}\right)$$

$$= \lim(1-a)\left[1 - \frac{1-a}{n}\right]^{-1}$$

$$= \lim(1-a)\left[1 + \frac{(1-a)}{n} + \left(\frac{1-a}{n}\right)^2\frac{1}{2!} + \cdots\right]$$

$= 1-a$

when $1-a > 1$

given series is ~~divergent~~ convergent.

when $1-a<1$ then div.

when $1-a=1$ or $a=0$, test fails.

(finite & unique) $S=1$, $\lim S_n = S$

In this case given series reduces to single term, namely 1. $\therefore$ convergent

Ques) test for convergence of the series $1+\frac{1}{2}\cdot\frac{x^2}{4}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\cdot\frac{x^4}{8}+\frac{1\cdot3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8\cdot10}\cdot\frac{x^6}{12}+\ldots\ldots$

Soln:-

$$u_n = \frac{1\cdot3\cdot5\ldots(4n-3)\cdot x^{2n}}{2\cdot4\cdot6\ldots(4n-2)\cdot 4n}$$

$$u_{n+1} = \frac{1\cdot3\cdot5\ldots(4n+1)\cdot x^{2(n+1)}}{2\cdot4\cdot6\ldots(4n+2)\cdot 4(n+1)}$$

$$\frac{u_n}{u_{n+1}} = \frac{[1\cdot3\cdot5\ldots(4n-3)\cdot x^{2n}]\,[2\cdot4\cdot6\ldots(4n-2)(4n+2)\cdot4(n+1)]}{[2\cdot4\cdot6\ldots(4n-2)\cdot4n]\,[1\cdot3\cdot5\ldots(4n+1)\cdot x^{2n+2}]}$$

$x^{2n}\cdot x^2$

$$= \frac{1}{x^2}\,\frac{(4n+2)(4n+4)}{(4n-1)(4n+1)} = \lim\frac{u_n}{u_{n+1}} = 1/x^2$$

when $1/x^2>1$ (conv)

" $1/x^2<1$ (div)

" $x^2=1$ (test fails)

$$\frac{u_n}{u_{n+1}} = \frac{(4n+2)(4n+4)}{(4n-1)(4n+1)} = \frac{16n^2+16n+8n+8}{16n^2+4n-4n-1}$$

$$= \frac{16n^2+24n+8}{16n^2-1}$$