5/8/22

lecture-5

Order Description Table :-

① $(U(1), \cdot_1)$ $\quad \phi(U(1)) = 1$

$U(1) = \{1\}$

Element	Order
1	1

② $(U(2), \cdot_2)$ $\quad |U(2)| = 1$

$U(2) = \{1\}$

Element	Order
1	1

③ $(U(3), \cdot_3)$ $\quad |U(3)| = 2$

$U(3) = \{1, 2\}$

Element	Order
1	1
2	2

④ $(U(4), \cdot_4)$ $\quad |U(4)| = 2$

$U(4) = \{1, 3\}$

Element	Order
1	1
3	2

⑤ $(U(12), \cdot_{12})$

$U(12) = \{1, 3, 5, 7\}$

$$\phi(12) = \phi(2^2 \cdot 3)$$
$$= 12\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)$$
$$= 12 \times \frac{1}{2} \times \frac{2}{3} = 4$$

Element	Order
1	1
3	2
5	2
7	2

Note:- Two groups $(G_1, *)$ and $(G_2, *)$ are isomorphic say $G_1 \cong G_2$ if they both have same Order Description Table.

Ex → $(U(1), \cdot_1)$ and $(U(2), \cdot_2)$ are isomorphic

Remark:- If G_1 is isomorphic to G_2 then they both have same properties i.e. If G_1 is cyclic $\Rightarrow G_2$ is cyclic and vice-versa.

Show That → $(U(8), \cdot_8)$ is an abelian group

$U(8) = \{1, 3, 5, 7\}$

$\cdot_8$	1	3	5	7
$\boxed{1}$	$\boxed{1}$	3	5	7 ✓
3	3	$\boxed{1}$	7	5
5	5	7	$\boxed{1}$	3
7	7	5	3	$\boxed{1}$

Since, all the elements under Caley Table belongs to the given set, thus it satisfies closure and associative property.

Here $e = 1$

Thus, it satisfies existence of identity.

$(1)^{-1} = 1$

$(3)^{-1} = 3$

$(5)^{-1} = 5$

$(7)^{-1} = 7$

Since, inverse of all the elements exists.
Hence, it satisfies existence of inverse.

Since, the matrix under Caley Table is symmetric in nature. Thus, it satisfies commutative property.

Therefore, $(U(8), \cdot_8)$ is an abelian group.