/11/22 convergent.

Q. Test the following series for absolute convergence $\rightarrow \frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$

$$\Sigma |u_n| = \frac{2}{1^2} + \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$$

$$|u_n| = \frac{n+1}{n^2}$$

$$\cancel{\lim |u_n| = \lim \left(\frac{1}{n} + \frac{1}{n^2}\right)} \quad |u_n| = \frac{\left(1 + \frac{1}{n}\right)}{n}$$

$$v_n = \frac{1}{n}$$

$$\lim \frac{u_n}{v_n} = \lim \left(1 + \frac{1}{n}\right) = 1 \text{ (non-zero finite no.)}$$

$\Sigma v_n = \Sigma \frac{1}{n}$, which is divergent

By comparison test $\Sigma |u_n|$ is also divergent.
Since $\Sigma |u_n|$ is not convergent.
Therefore, Σu_n is not absolutely convergent.

Q Show that the series $\sum \frac{\sin n\theta}{n^2}$ and $\sum \frac{\cos n\theta}{n^2}$ are absolutely convergent.

$\sum \left|\frac{\sin n\theta}{n^2}\right| \leq \sum \frac{1}{n^2}$, which is convergent by p-series test.

Therefore, $\sum \left|\frac{\sin n\theta}{n^2}\right|$ is also convergent.

Hence, $\sum \frac{\sin n\theta}{n^2}$ is absolutely convergent.

Similarly, $\sum \left|\frac{\cos n\theta}{n^2}\right| \leq \sum \frac{1}{n^2}$

$\Rightarrow \sum \left|\frac{\cos n\theta}{n^2}\right|$ is convergent.

$\Rightarrow \sum \frac{\cos n\theta}{n^2}$ is absolutely convergent.

Rearrangement of Series:

If the terms of an absolutely convergent series are rearranged the series remains convergent and its sum is unaltered.

Eg → let $S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$

& $S' = \left(1 - \frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{6} - \frac{1}{8}\right) + \ldots$ ①

$S' = \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{6} - \frac{1}{8}\right) + \left(\frac{1}{10} - \frac{1}{12}\right) + \ldots$ ②