Ques → Rearrange the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ in such a manner that the sum of rearranged series is $\frac{3}{2} \log 2$.

Solu →
$$\frac{1}{2} \log 4k = \frac{3}{2} \log 2$$
$$\log 4k = 3 \log 2$$
$$\log 4k = \log 8$$
$$k = 2$$
$$\Rightarrow \frac{m}{n} = \frac{2}{1}$$
$$\Rightarrow m = 2, \ \& \ n = 1$$

∴ rearrange series is

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \dots$$

Cauchy's general principle for convergence of a series :→

A necessary and sufficient condition for the convergence of series Σu_n is that corresponding to arbitrary chosen number $\varepsilon > 0$, there exist a positive integer N such that

$$\left| \sum_{\nu=N+1}^{N+p} u_\nu \right| < \varepsilon, \quad \text{for all positive integral value of } p \text{ and } n \geq N$$

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Limit Superior and limit Inferior :-

Let $\{x_n\}$ be a bounded sequence of real no..
Let M_1 be the least upper bound of $\{x_n\}$; — ①
Let M_2 be the lub of $x_2, x_3, x_4, \ldots\ldots$ — ②
⋮
Let M_n be the lub of $x_n, x_{n+1}, x_{n+2}, \ldots\ldots$ — ⓝ
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Similarly, let $m_1, m_2, \ldots\ldots m_n, \ldots\ldots$ be the glb of eq ①, ②, ... ⓝ, ... respectively.

Clearly,

$$M_1 \geq M_2 \geq M_3 \geq \ldots\ldots \geq m_1$$

$\because$ The sequence of $\{M_n\}$ is monotonic decreasing and bounded below.

$\therefore$ $\lim M_n$ exist and is finite.

Let $\lim\limits_{n\to\infty} M_n = \Delta$, then $M_n \geq \Delta$, $\forall n$ — (i)

for $\varepsilon > 0$, $\exists$ a member M_ν of $\{M_n\}$ such that $M_\nu < \Delta + \varepsilon$ — (ii)