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Ques. Show that a constant fn k is integrable and

$$\int_a^b k\,dx = k(b-a)$$

Solns. For any partition P of the $[a,b]$, we have

$P = \{a = x_0, x_1, x_2, \dots, x_{i-1}, x_i, \dots x_n = b\}$

Define $\delta_i = x_i - x_{i-1}$

For integrable, we have to show that

$$\underline{\int_a^b} f\,dx = \overline{\int_a^b} f\,dx$$

$$L[P,f] = \sum_{i=1}^{n} m_i \delta_i$$

$$= k \sum_{i=1}^{n} \delta_i$$

$$= k[\cancel{x_1} - \underset{a}{x_0} + \cancel{x_2} - \cancel{x_1} + \dots \cancel{} + \underset{b}{x_n} - \cancel{x_{n-1}}]$$

$$= k(b-a)$$

$$\underline{\int_a^b} f\,dx = \sup L[P,f]$$

$$= k(b-a) \quad \text{——— ①}$$

$$U[P,f] = \sum_{i=1}^{n} M_i \delta_i$$

$$= k(b-a)$$

$$\overline{\int_a^b} f\,dx = \inf U[P,f]$$

$$= k(b-a) \quad \text{——— ②}$$

from ① & ② eqn

$$\underline{\int_a^b} k\,dx = \overline{\int_a^b} k\,dx = k(b-a) = \int_a^b k\,dx$$

Proved

Ques → Show that the function f defined by

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable on any interval.

Solution → $L[P,f] = \sum_{i=1}^{n} m_i \delta_i$

$= m_1\delta_1 + m_2\delta_2 + \cdots + m_n\delta_n$

$= 0\cdot\delta_1 + 0\cdot\delta_2 + \cdots + 0\cdot\delta_n$

$= 0$

$$\underline{\int_a^b} f\,dx = \sup L[P,f] = 0 \quad \text{--- (1)}$$

$U[P,f] = \sum_{i=1}^{n} M_i \delta_i$

$= M_1\delta_1 + M_2\delta_2 + \cdots + M_n\delta_n$

$= \delta_1 + \delta_2 + \cdots + \delta_n$

$= b-a$

$$\overline{\int_a^b} f\,dx = \inf U[P,f] = b-a \quad \text{--- (2)}$$

Since, $\underline{\int_a^b} f\,dx \neq \overline{\int_a^b} f\,dx$

Hence, f is not R-integrable

Ques → Show that x^2 is integrable on any interval $[0, k]$.