8/8/22

Lecture-6

Polygon :- Polygon is a simple closed geometry with atleast 3 straight lines.

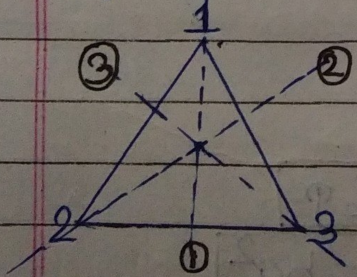
Regular Polygon :- Regular Polygon is a polygon with all its sides as well as angles equal.

Symmetry :- An undetectable motion is called symmetry.

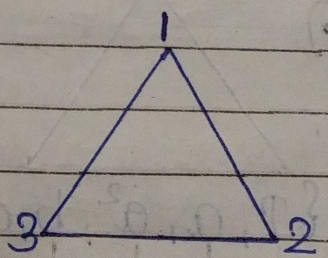
There are two kinds of symmetries —
1. Reflexive Symmetry
2. Rotational Symmetry

1. Reflexive Symmetry :-

Case I → If n = odd no. (3, 5, ----)
the symmetric lines will be obtained by joining each vertex to the midpoint of the opposite sides.

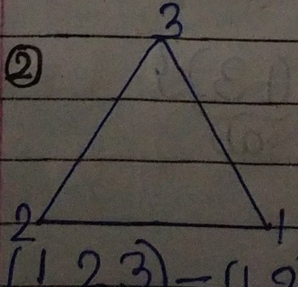


①



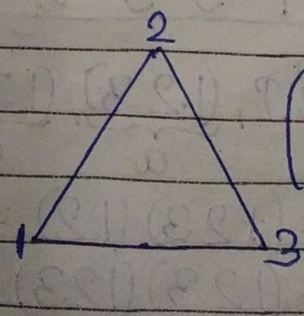
$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}=(2\,3)$

②



$\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}=(1\,3)$

③



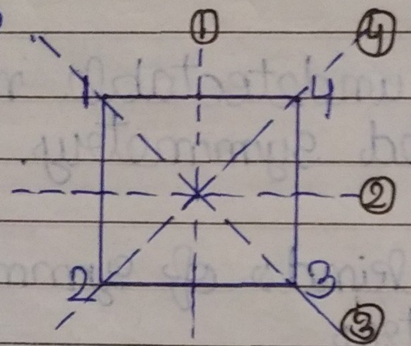
$\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}=(1\,2)$

(ii) Case II → If n = even no. (4, 6, ...)

no. of reflexive symmetries $= \frac{n}{2} + \frac{n}{2} = n$

$n/2$ are obtained by joining midpoint of opposite sides and

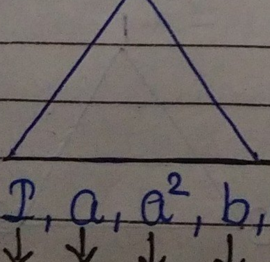
other $n/2$ are obtained by joining opposite vertices



Dihedral Group (D_n, *) :- Collection of all reflexive and rotational symmetries corresponding to a n-sided regular polygon with binary composition is called dihedral group.

$$\boxed{O(D_n) = 2n}$$

Ex (D_3, ·)



$$\boxed{\begin{array}{l} D_3 = \{I,\ a,\ a^2,\ b,\ ab,\ a^2b\} \\ \quad\ \ \downarrow\ \ \downarrow\ \ \ \downarrow\ \ \ \downarrow\ \ \ \downarrow\ \ \ \ \ \downarrow \\ \quad\ \ 1\ \ \ 3\ \ \ 3\ \ \ 2\ \ \ 2\ \ \ \ \ 2 \end{array}}$$

$\boxed{a^3 = I}$

$\boxed{ab = ba^2}$

$D_3 = \{I, \underbrace{(1\,2\,3)}_{a}, \underbrace{(1\,3\,2)}_{a^2}, \underbrace{(1\,2)}_{b}, \underbrace{(2\,3)}_{a^2b}, \underbrace{(1\,3)}_{ab}\}$

$a \cdot b = (1\,2\,3)(1\,2) = (1\,3)$

$a \cdot a = (1\,2\,3)(1\,2\,3) = (1\,3\,2)$

$a^2b = (1\,3\,2)(1\,2) = (2\,3)$

Order Description Table →

Element	Order
I	1
a	3
a^2	3
b	2
ab	2
a^2b	2

$D_4 = \{I, a, a^2, a^3, b, ab, a^2b, a^3b\}$

$a^4 = I$

$ab = ba^3$

$o(D_4) = 8$

$$\boxed{D_n = \{\underset{1}{\underset{\downarrow}{I}}, \underset{n}{\underset{\downarrow}{a}}, a^2, \ldots a^{n-1}, \underset{2}{\underset{\downarrow}{b}}, \underset{2}{\underset{\downarrow}{ab}}, \underset{2}{\underset{\downarrow}{a^2b}}, \ldots \underset{2}{\underset{\downarrow}{a^{n-1}b}}\}}$$

where, $a^n = I$

$ab = ba^{n-1}$

Now, $\boxed{G_n = \{I, a, a^2, \ldots a^{n-1}\}}$

* $(G_n, \cdot)$ is a cyclic group of rotations.

* $(D_n, \cdot)$ will never be cyclic.

* Dihedral group (D_n) is always a non-abelian group of order $2n$ where $n \geq 3$

Caley Table → $(D_3, \cdot)$

$\cdot$	I	a	a^2	b	ab	a^2b
I	$\boxed{I}$	a	a^2	b	ab	a^2b
a	a	a^2	$\boxed{I}$	ab	a^2b	a^3b
a^2	a^2	a^3	$\boxed{I}$	a^2b	b	ab
b	b	a^2b	ab	$\boxed{I}$	a^2	a
ab	ab	b	a^2b	a	$\boxed{I}$	a^2
a^2b	a^2b	ab	b	a^2	a	$\boxed{I}$

$ab = ba^2$ $\qquad$ $a^3 = I$

$aba^{-1} = ba(aa^{-1})$ $\qquad$ $a^2 = a^{-1}$

$ab a^2 = ba$

$a^2 b = ba$

* If G is a cyclic group of order n and d/n then no of elements of order 'd' in $G = \phi(d)$