20/8/22 Proved

Neighbourhood of a point:- If $p \in R$ and $\epsilon > 0$, then the open interval $(p-\epsilon, p+\epsilon)$ is called an ϵ-neighbourhood of p and is denoted by $N_\epsilon(p)$. A neighbourhood of p is any set which contains an ϵ-neighbourhood of p for some $\epsilon > 0$.

If N is the nbd of p, then the set $N - \{p\}$ is called a deleted nbd of p.

Ex-① If $I = (2,5)$ then I is a nbd of a point $4 \in (2,5)$ because $\epsilon = 1/2$ such that ——

$$\left(4-\frac{1}{2}, 4+\frac{1}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right) \in (2,5)$$

Ex-② The set R of real number is a nbd of each of its points.

Ex-③ The set Q of rational numbers is not a nbd of any of its point.

Theorem:- 'Every open interval is a nbd of each of its points'

Proof → Let (a,b) be an open interval and p be any point of (a,b). We have $p-a>0$ and $b-p>0$. Let us choose →

$\epsilon = \min\{p-a, b-p\}$, so that $\epsilon>0$

We have to show that $(p-\epsilon, p+\epsilon) \subseteq (a,b)$

Let $x \in (p-\epsilon, p+\epsilon)$

$\Rightarrow p-\epsilon < x < p+\epsilon$

$-\epsilon < x-p < \epsilon$

Since, $\epsilon \le p-a$ i.e. $a-p \le -\epsilon$ and $\epsilon \le b-p$

it follows that → $a-p < x-p < b-p$ i.e

$a < x < b$

Hence, $(p-\epsilon, p+\epsilon) \subseteq (a,b)$

—— Proved