16/8/22

lecture-7

Properties of Groups:-

① Theorem:→ 'Let G be a group. G has unique identity element'

Proof → Suppose G has 2 identity elements say e and e'

$\Rightarrow e \in G$ and $e' \in G$ $(e \neq e')$

$e * a = a * e = a \quad \forall\, a \in G$ —— ①

$e' * a = a * e' = a \quad \forall\, a \in G$ —— ②

from eq ① take $a = e'$

$e * e' = e' * e = e'$ —— ③

from eq ② take $a = e$

$e' * e = e * e' = e$ —— ④

from eq ③ & ④, we get →

$e = e'$

It contradicts our assumption that $e \neq e'$

Hence, $e = e'$

Therefore, group G has unique identity element.

—— Proved

② **Cancellation Law:-** Let G be a group & $a,b,c \in G$ s.t. $a*b = a*c$ then $b=c$

Proof→ $a \in G \Rightarrow a^{-1}$ exists

$\Rightarrow a^{-1}*a = e = a^{+}*a^{-1} \quad \forall\, a \in G$

Now $a*b = a*c$

$(a^{-1}*a)*b = (a^{-1}*a)*c$

$b = c$

——— Proved

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③ Let G be a group then each element 'a' in G has unique inverse in G.

Proof→ Let element 'a' in G has 2 inverse say b and b'

$\Rightarrow b, b' \in G \quad (b \neq b')$

By definition of inverse ——

$a*b = e$ —— ①

$a*b' = e$ —— ②

from eq ① & ②, we get ⟶

$a*b = a*b'$

By left Cancellation law, we get ⟶

$b = b'$

It contradicts our assumption that $b \neq b'$

Hence, $b = b'$

Therefore, each element 'a' in group G has unique inverse in G.

——— Proved