17/8/22

Lecture-8

Unit-4

Permutation Group

$f: A \to B$

$|A| = m$, $|B| = n$

one-one $f^n \to \begin{cases} {}^nP_m & \text{if } n \geq m \\ 0 & \text{if } m > n \end{cases}$

Permutation Group → denoted by S_n or P_n

S_n : Symmetric group of degree 'n' on n symbols

P_n : Permutation group of degree 'n' on n symbols.

S_n = collection of all one-one and onto f^n from X to X where $X = \{1, 2, 3, \ldots n\}$

$S_n = \{f: X \to X, f \text{ is one-one onto}\}$

$|S_n| = n!$ or $|P_n| = n!$

$|S_3| = 3! = 6$

$|S_4| = 4! = 24$

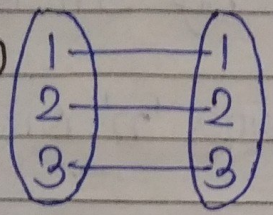
Q. Write all the elements of S_3.

$S_3 = \{f : \{1,2,3\} \to \{1,2,3\}, f \text{ is one-one onto}\}$

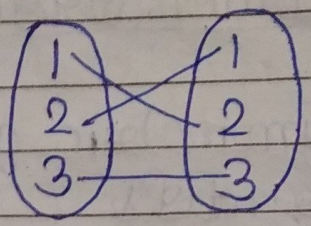
$|S_3| = 3! = 6$

Order of elements of S_3 = divisors of 6
= 1/2/3/6

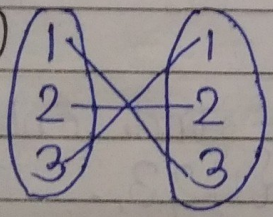
(I) ①



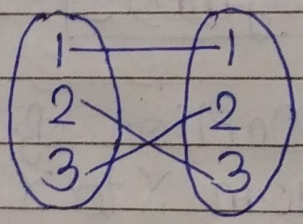
②



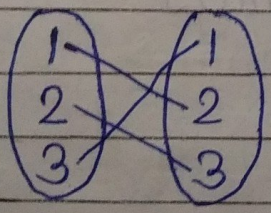
③



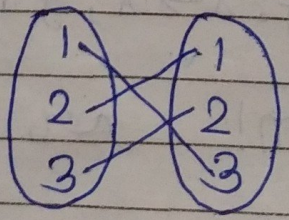
④



⑤



⑥



(II) ① $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

② $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

③ $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

④ $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

⑤ $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

⑥ $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

(III) (1)(2)(3) , (1 2)(3) , (1 3)(2)
(1)(2 3) , (1 2 3) , (1 3 2)

(IV) I, (1 2), (1 3), (2 3), (1 2 3), (1 3 2)

$S_3 = \{I, (1\,2), (1\,3), (2\,3), (1\,2\,3), (1\,3\,2)\}$
↓ 1 ↓ 2 ↓ 2 ↓ 2 ↓ 3 ↓ 3

* $(S_3, \cdot)$ is a non-cyclic group.

Closure Property →

(1 2)(1 3) = ~~(1 3) (2)~~ (1 3 2)
(1 3)(2 3) = ~~(1 3) (2)~~ (1 3 2)
(1 2)(2 3) = (2 3)
(1 2)(1 3 2) = (1 3)
(1 3)(1 2) = (1 2 3)
(1 3)(1 2 3) = (1 2)